MINISTRY OF
EDUCATION

THE NATIONAL COMPREHENSIVE NUMERACY PROGRAMME

ACKNOWLEDGEMENTS

The purpose of this document is to provide guidance and support to teachers from the Early Childhood level through to Grade 6 in the appropriate and effective way to prepare children and pupils to become numerate. The National Comprehensive Numeracy Programme with this document, therefore, will provide the context and approach to be employed in mathematics instruction throughout these levels; so as to set the right foundation upon which to develop the skills and competencies needed for students to be successful in mathematics, and to display adequate and appropriate numeracy skills in their everyday life.

The document establishes the approach to be employed which is founded on three fundamental principles:

1. Conceptual Understanding
2. Computational Fluency
3. Problem Solving

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FOREWORD

It is accepted that numeracy is a basic human right of the 21st century citizen. It is through the teaching of mathematics that an individual's numeracy skills are developed. As a result, mathematics has been universally accepted as a core subject, occupying a central place in education systems worldwide.   This is because it plays an important role in helping an individual learn to reason logically and think critically – skills which are important to other disciplines and situations in real life. Taught well, mathematics can engage students, and help them understand how the world works while exposing them to some of its unanswered mysteries.[1]

There are several definitions for numeracy; in this document we will give focus to the following:

- *Numeracy is concerned with using, communicating and making sense of mathematics in a range of everyday applications; the ability to explore, hypothesize and reason logically and to use a variety of methods to solve problems.*

- *Numeracy is the ability to reason with numbers and other mathematical concepts. To be numerically literate, a person has to be comfortable with logic and reasoning.*

- *Numeracy is a proficiency which is developed mainly in mathematics, but also in other subjects. It is more than an ability to do basic arithmetic. It involves developing confidence and competence with numbers and measures.*

Words help us paint a picture of the world, but numbers and mathematics give us the tools we need to understand how the world and the universe work.

The Mandate
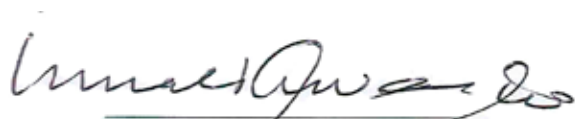
In recognition of this important connection between mathematics, numeracy and the growth and development of an individual and the nation, the Ministry of Education has set a target of 85% mastery in numeracy for all students at Grade 4 by 2015. The Minister of Education and the Permanent Secretary shall be responsible for ensuring that the conditions that will enable the

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1   http://www.learningfirst.org/

achievement of this target are in place. The National Comprehensive Numeracy Programme for Early Childhood and Grades 1– 6, along with a number of initiatives and resources, are a part of the Ministry's effort to ensure that the target is achieved, an accomplishment whose added benefit will be improvement in numeracy performance at the Primary level. One critical component of the Ministry's programme is the provision of strong and targeted support from Numeracy Specialists. The support of the Specialists, complemented by good health practices, the synchronised efforts of parents and teachers, school administrators and the proper assessment and diagnosis of pupils with special needs, will auger well for the achievement of 85% mastery in numeracy.

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Chapter 1:
Contextual Framework

Introduction

Mathematics is a well-established discipline, universally accepted as a core subject, occupying a central place in education. It is taught from the earliest grades, and provides a foundation for learning most other disciplines. Mathematics has its own inescapable beauty and when taught well can engage and delight students, helping them in their understanding of how the world works, while at the same time exposing them to some of its unanswered mysteries. Mathematics teaches students how to reason logically, helps them develop critical thinking skills and provides them with the tools required to function effectively in their environment.

Making use of mathematics as a means of developing critical thinking, problem solving skills and innovativeness remains an elusive goal. For this to be achieved, mathematics would need to be taught, especially in the early years, in a manner that makes it developmentally appropriate to learners in terms of their individual readiness, cultural background, and emotional profile. The nature of the subject requires that when it is used to assess students' development, it is done in a manner to endorse the ability of students to accept and rely on it in their daily transactions.

To keep pace with the demands of the ever increasing information, communication, technological age, a renewed energy has been devoted to mathematics. This renewed interest in the utility of mathematics has contributed to an unprecedented reliance on the discipline in all spheres of human endeavour. It is this societal demand on mathematics that has given birth to numeracy. Numeracy is a measure of an individual's competence in mathematics and his/her ability to apply the concepts learned in everyday life. Akin to literacy, it enhances students' understanding of all subjects and their capacity to lead informed lives. Numeracy is not the same

as mathematics, nor is it an alternative to mathematics but comes out of mathematics. It is an equal and supporting partner in helping students learn to cope with the quantitative demands of modern society. The ever present need for numeracy is somewhat new, and as a result of the public's increased recognition of its pervasive role in daily life and work, greater demand is now being made on education systems to create 'the numerate person'. Numeracy must therefore permeate the curriculum.

Numeracy is the ability to apply mathematics to solving real life problems. It is not so much about understanding abstract concepts as it is about applying elementary tools to everyday situations. It is important in this discussion to consider what has fuelled this need to focus on numeracy development. The increased focus on numeracy development is one of the results of the vast transformation which has taken place in the world's economies, led by an increased focus on technology driven sectors, along with the propensity to use numbers to support arguments of all kinds in the realms of politics, economics, social reform and other 'new' areas of life in the 20th and 21st centuries. This includes taking censuses, evaluating medical outcomes using simple statistics, and compiling numerical facts about national development.

Of the several definitions of numeracy posited, the following has been adopted as the framework which will influence the outcomes to be achieved under the National Comprehensive Numeracy Plan:

> *Numeracy, like literacy, provides key skills to enable children and young people to be successful at school and life beyond it. Numeracy is a prerequisite for success in a changing society where people need to be ready to adapt to new jobs, new career pathways and new technologies. To be numerate is to use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community and civic life. In school education, numeracy is a fundamental component of learning, discourse and critique across all areas of the curriculum. It involves the disposition to use, in context, a combination of:*

> - *Underpinning mathematical concepts and skills from across the discipline (numerical, spatial, graphical, statistical and algebraic);*
> - *Mathematical thinking and strategies;*
> - *General thinking skills; and*
> - *Grounded appreciation of context.*

The Jamaican Context

In Jamaica there is concern about the unsatisfactory performance of students in mathematics at all levels of the education system. The concern surrounding the low levels of performance stems from the realization that they are an indicator that an insufficient number of persons in the society are equipped with the skills and understandings required to function effectively in life and to apply the mathematics they have learnt to unfamiliar contexts. One significant contributing factor to the low levels of performance is the poor attitudes to the subject of many students and teachers. The poor attitude of students is often compounded by the view some students hold that the subject is of little use to them outside of school, particularly since it is usually taught with a high level of abstraction – heightening the problems caused by the disconnect the society has between mathematics and numeracy. Abstraction inhibits the ability of students to develop an awareness of the applicability of mathematics concepts and ideas to their everyday lives and experiences. It is our ability to apply these mathematical principles to our daily living experiences that enables our successful navigation of the simplest to the most complex activities of our lives – whether reading a clock, bank statement or utility bill or understanding a statistical chart. The fact that there have been no commonly agreed principles, aims and objectives to date for mathematics education in Jamaica, has only served to exacerbate the problem.

Another significant contributing factor to the low levels of student performance in mathematics stems from the way in which students are engaged in learning the subject in the classroom. There is now a growing debate among educators concerning this particular matter. In other words, the challenge is that mathematics is often presented as an isolated stand-alone discipline which is taught through procedures and rules which are to be followed.

The method which should be employed in the teaching of mathematics has formed a critical basis in the development of the National Comprehensive Numeracy Programme. The approach is espoused at the early childhood and primary levels through the national standardised curricula, which places heavy reliance on an approach to teaching and learning of mathematics which promotes the development of conceptual understanding, computational fluency and problem solving skills. These three components of sound mathematics teaching are critical to an individual's ability to attain mastery in numeracy and they are best developed in a classroom context which is student-centred and in which there is a consistent effort to practically integrate mathematical ideas with other subject areas.

The idea of integration is strongly promoted through the Revised Primary Curriculum (RPC) which was written with the recognition of the importance of mathematics to primary education

- particularly to the development of numeracy skills. Deliberate efforts are made in the design of the RPC to promote the integration of mathematics ideas with other subject areas. But this is balanced by the requirement that teachers give focus to the teaching of mathematics during the daily one hour "Mathematics Window".

Over the years, there have been many steps taken with a view to improve awareness and value for mathematics by both the government and private sector. However these efforts have not been sustained. These projects include:

- PEIP I & II (Primary Education Improvement Project) that developed the Revised Primary Curriculum (RPC) in 1999.

- PESP (Primary Education Support Project) which began in 2001 and was designed to implement the Revised Primary Curriculum (RPC).

- NHP (New Horizons Project) which worked with 72 schools.

- JAASP (Jamaica All-Age Schools Project) which worked with 48 schools.

Projects such as NHP and the JAASP have made serious attempts at addressing numeracy through the development of the country's first National Numeracy Policy. Both projects, however, ended before the implementation of the National Numeracy Strategy.

The Ministry of Education, through the National Comprehensive Numeracy Programme, intends to promote its philosophy of mathematics teaching and learning so as to change the cultural attitude that exists. This path places strong emphasis on early mathematics education. The objective of this approach is not only to change the student's mindset, but ultimately spark a greater appreciation for numbers and the relevance they play in our lives.

Chapter 2:

Promoting Numeracy

Raising public awareness of the importance of mathematics is a very important task for every school. This is because there is a need to raise the level of awareness among members of the society of the role that mathematics plays in the development of numeracy, particularly since it is a critical foundation for many scientific and technological advancements.

Since numeracy is the fundamental cornerstone in many diverse areas of society, it is important that its development through the teaching and learning of mathematics is promoted at all levels, not only by schools, but also by policy makers and other stakeholders. The focus must be that every child should develop numeracy skills during his/her time at school. A numerate individual is one who has the confidence and ability to choose and use mathematics skills learnt in everyday life.

Numeracy can be promoted through:
- **Deliberately promoting an appreciation for mathematics**
- **Strategic school leadership**
- **Constructive parental involvement**

In this section of the document we will examine strategies related to promoting an appreciation for mathematics, the provision of strategic leadership and parental involvement. Effective curriculum delivery will be addressed in greater detail in subsequent chapters.

Promoting an Appreciation for Mathematics

To raise the standard of numeracy, the mathematics curriculum must be delivered in a manner which promotes an appreciation for the subject and helps students see its link to our everyday life and experiences. From the early years, children need to be cognizant of the importance

of mathematics as a life skill and must be able to develop the necessary competencies and understanding that will assist them to be numerate. Once students can make the connection between mathematics and their real life experiences, there is the likelihood that an appreciation for the subject will be developed. Where an appreciation of mathematics exists, numeracy development will most likely be impacted in a positive way, even for students who may find the subject challenging.

An approach to teaching and learning which consistently links mathematical ideas to our everyday life and experiences is one way in which we can make learning the subject more interesting for students. This will mean that teachers will have to think about identifying activities that demonstrate the link between what is being taught and the current or future everyday experiences of their students.

Numeracy Based Activities Related to the Home

Everyday activities which can be explored and discussed in the teaching of critical mathematics concepts include, but are not limited to, those surrounding:

- **Balancing a Budget:** This can afford opportunities for students to see mathematics at work in the way in which their parents balance their monthly budget. Students can be engaged in a similar activity, where they are assigned a hypothetical budget scenario which includes information such as occupation, family size, type of home and amount of mortgage or rent to be paid.

- **Shopping**: Activities centred on shopping can help students to solve problems which involve making change, determining what items can be purchased given a specific amount of money, and choosing the right denominations of money in a purchasing scenario. In addition, they will appreciate that discounts and taxes can impact their spending power.

- **Food Preparation:** Activities carried out in the kitchen allow students to understand the relationship between and among different quantities. For example, in order to create dishes with a good flavour, various ingredients must be used in the correct proportions. Recipes require knowledge of whole numbers as well as fractions, and are a good base for helping children to understand ratio. If a recipe calls for 1 egg and 2 cups of flour, the relationship of eggs to cups of flour is 1 to 2.This is particularly important in instances when, for example, one needs to prepare an item for 12 persons, but the recipe gives quantities for catering for 4 persons.

For other examples of activities that promote an appreciation for numeracy, see the handbook *Putting Mathematics into Family Life*.

Numeracy Based Activities Related to School

Allowing children to experience mathematics in ways that demonstrate its applicability to their daily lives is just one critical strategy for improving their appreciation for the subject. There are other effective practices which can contribute significantly to students gaining interest in mathematics, many of which can be employed from the early childhood and lower primary years. This includes creating a mathematics-rich classroom with a wide array of materials for students to explore and manipulate. There should be a playful mathematics atmosphere where students are consistently able to be immersed in mathematics games and activities at their own leisure. Children are unaware that during play and daily activities they often explore mathematical ideas and processes ("He has more than I do." "I have less."). In addition to creating opportunities for play and exploration in the mathematics classroom, it is also important that we provide the opportunity for students to think about mathematics ideas, and this is facilitated through the questions that can be asked both during the teaching of a lesson and in interacting with them in everyday circumstances (both at home and at school). Questions such as, "How should we sort the fudge sticks?" "How can we divide the fudge sticks so each one gets the same?" encourage students to think and reason mathematically and see the connections between what they have learnt and the everyday experiences they often have.

Effective practice does not limit mathematics to one specified period or time of day. Rather, teachers should help children develop mathematical knowledge throughout the school day, especially in the early years. For example, when students are lining up, teachers can build in many opportunities to develop an understanding of mathematics:

- o Students wearing something red can be asked to get in line first;
- o Those wearing blue can get in line second, and so on;
- o Students wearing something red as well as sneakers can be asked to go to the top of the line, and so on.

Opportunities like these which help to build important mathematical vocabulary and concepts abound inside and outside of the classroom, and the alert teacher should take full advantage of them.

Strategic School Leadership

Numeracy develops and evolves through experiences that require students to creatively and flexibly use mathematics to solve complex problems (*ERIC Digest*). We need to make allowances for the type of experiences that will allow children to think. This has been a challenge in primary schools because of the pressures of a tightly packed mathematics curriculum, adequate time is often not given to ensuring the development of conceptual understanding. This is often the case because of the emphasis on preparing students for high-stakes tests, which on their own are not able to measure numeracy development. While these national tests are important, it is even more important that the leadership of schools facilitate students' numeracy development by creating a school culture that emphasizes the importance of numeracy. Principals, as instructional leaders, should advocate for and support classroom environments that have mathematical resources and activities which support effective delivery of the mathematics curriculum and facilitate creativity and flexibility in mathematical thinking. A Principal, who maintains a commitment to numeracy as part of the effort to ensure that numeracy development is a school goal, provides leadership in the implementation of strategic activities which include but are not limited to:

- Supporting the on-going professional development of teachers through the provision of staff development sessions that are geared towards teaching of mathematics so that teachers feel empowered to better deliver the mathematics content to students. It is recommended that schools hold monthly professional development sessions and seek support through their Quality Education Circle (QEC);

- Ensuring that there are consistent common planning sessions.

Therefore, a school that promotes numeracy must exhibit the following:
- Ensuring that teachers plan for the differentiated classroom and are therefore prepared to cater to students at different levels and with varying learning styles;

- Developing 'mathematics–rich' classrooms which are equipped with problem solving games and activities which can stimulate students' interest in learning and using mathematical concepts;

- Ensuring that lessons which are planned and delivered are designed to:

  a) develop concepts rather than focus on procedures. The use of manipulatives is critical to the development of concepts.
  b) link mathematics to other subject disciplines so that students are able to see the connections between mathematics in other subject areas.

c) promote discussions which allow student to use mathematical terms and concepts. This can be done by ensuring that effective questions are consistently outlined in lesson plans designed to guide the delivery of the mathematics curriculum. The use of discussions guided by effective questions is a powerful method through which students can develop fluency in the use and understanding of mathematics ideas.

d) link concepts taught to everyday situations, both in the identification and use of activities inside and outside (e.g. field trips) of the mathematics classroom.

- Providing guidance for teachers so that they are able to effectively use the teaching and learning of mathematics to foster the development of the critical thinking skills and creativity of their students ;

- Promoting and developing Math Centres/Corners in as many classrooms as possible within the school;

- Employing alternative assessment strategies in the classroom;

- Encouraging the involvement of parents in the students' learning of mathematics.

In addition to the activities highlighted above, the Principal can also ensure that numeracy is a focus in the life of the school by incorporating and engaging students in a range of activities outside of the mathematics classroom which, while promoting numeracy development, have the added benefit of improving the attitudes of students and teachers. The frequency of these activities may vary. They include but are not limited to:

Establishing Mathematics Clubs: The activities of a club have the potential to get students excited about mathematics. They should include, free discussions, games, the solution of puzzles, competitions and other activities which DO NOT take place in the regular classroom situation. Examples of activities that can be used in a Mathematics Club can be found in the *Mathematics Interest Building Handbook*.

Math Fun Days that are themed: Here the focus of that day allows students to pay attention to the concept being highlighted such as 'Metric Day', 'Shapes Day', etc. Students should be involved in various activities that will help them to better understand the identified concept and as a result develop an appreciation for it.

Yearly Mathematics Competitions at various levels: (*See Mathematics Interest Building Handbook*): Celebrating Mathematics Month (this is usually done in April). Schools can identify one day in the month which will be celebrated as Mathematics Day. Here displays can be mounted or symposiums facilitated which provide an opportunity for students, parents, teachers

and other stakeholders to work together to strengthen mathematical competencies based on the identified theme.

Numeracy Leadership: The Role of the Numeracy Coordinator

The Numeracy Coordinator is able to provide consistent leadership for numeracy development in the life of the school. The Numeracy Coordinator can be given the responsibility for working closely with teachers and others vested with the responsibility of mathematics education, including the Numeracy Specialist. The Numeracy Coordinator should work directly with the Principal in planning and facilitating the activities outlined above. Other responsibilities should include:

- using his/her capacity to support professional development and common planning sessions for teachers;

- setting targets to improve numeracy development and student performance in mathematics. These targets should be clearly outlined in a Numeracy Plan which details the activities which will be used to ensure that they are met;

- ensuring that numeracy demands are met in other areas of the curriculum;

- knowing what is happening in the mathematics classroom and be able to speak to effective classroom practices by being a manager of the curriculum.

Intervention for Struggling Students

Another critical aspect of strategic school leadership is ensuring that provisions are made to offer remedial support for struggling students. In an effort to ensure that the needs of every learner are met, it is being proposed that a three-tier instructional model be used. This model, dubbed the **CTI (Core, Targeted and Intensive) Instructional Model** is built on the premise that struggling students should be identified and provided with instructional support suitable to their needs. Additionally, the model recognises that different levels of instruction are needed to respond appropriately to the needs of learners performing at different levels. The main elements of this instructional model are presented below:

- **First, there should be the creation of increasingly intensive teaching/learning opportunities to match students' needs.** All students should be exposed to the curriculum in a fairly standard manner. Where a student is assessed as needing additional support, targeted instruction should be provided. Students with marked difficulties and who have not

responded to additional instruction/support should be diagnosed and the appropriate programme developed to meet their needs. Diagnosis at the school level can take place through a critical analysis of the students' work to identify and interpret error patterns in an effort to determine if there are misconceptions, overgeneralisations or common errors.

- **Second, continuous assessment of students should be employed to identify their learning needs and/or evaluate their progress**. An initial assessment should be done at the start of the school year to determine the child's exposure to the subject. Further assessments should be done at the end of each unit in the mathematics curriculum to determine if students:
  - o have made adequate progress and no longer need intervention;
  - o continue to need some intervention;
  - o need even more intensive intervention.

- **Third, it is critical that instructional decisions be made at all tiers throughout the model;** these decisions should always be informed by data gathered from on-going assessment.

- **Fourth, efforts should be made to ensure that dynamic timetabling and human resource usage is able to facilitate the concurrent teaching of mathematics** (in small pull-out groups) and other subjects.

Elements of the Instruction Model

The CTI Instruction Model has three stages as outlined below.

Stage 1 – Core Instruction

- This instruction is offered daily to ALL students in the class for between 45 and 60 minutes. The quality of the instruction should be one in which the classroom teacher employs research-based effective practices to deliver sound mathematics lessons.

- Core instruction delivers the mathematics curriculum to the class as a whole. Strategies such as differentiated instruction and cooperative learning are expected to be used.

- During core instruction, the teacher will anticipate challenges students are likely to have with the content and will be proactive in implementing various initiatives to militate against students falling behind.

Stage 2 – Targeted Instruction

- This stage provides additional instruction for students in need of additional support. Additional support is provided to students through pooling of like abilities to deliver instruction. This requires timetabling to allow for common mathematics sessions across grades. In large schools, which have many classes per grades, one teacher could be assigned the weaker students while the other students in this teacher's class are distributed over the other classes. It is recommended that the strongest teacher be assigned the weakest grouping. In schools with only one class per grade or in multi-grade schools, the teacher should employ effective differentiated instructional strategies.

- So as to ensure that the instruction is targeted and specific to the needs of the least able students it is recommended that group sizes should be small.

- It is important to note that Stage 2 instructions are to be used to provide students with the skills necessary for staying apace with the rest of the class. The eventual aim must be for them to transition back into the core group when they are deemed to have made sufficient progress.

- In Stage 2 it is also important that lesson designs facilitate a high level of interaction, and:

  - focus on the development of number sense through games, music, dance, puzzles, etc.;
  - provide consistent attention to students developing fluency with and hence the ability to retrieve basic arithmetic facts;
  - integrate mathematics ideas and concepts with other subjects;
  - provide opportunities for field visits which can support students in making the connection between what they are learning and their everyday life experiences;
  - promote mathematical thinking by placing an increased emphasis on strategies and on verbalisation of thought processes;
  - provide more guided practice and corrective feedback;
  - facilitate continuous assessment.

Stage 3 – Individualised Instruction

- Students with marked difficulties and who have not responded to additional instruction/support (Stage 2) should be diagnosed and the appropriate programme developed to meet their needs. The classroom teacher should be supported by the school's Mathematics Specialist and where necessary with support of a Special Educator. Schools

are encouraged to make use of resources within their QEC's and the Regions to treat with children at Stage 3. It is recommended that the individualised learning plans should include, but not be limited to the following:

- o   focus on the development of number sense through games, music, dance, puzzles, etc.;

- o   integration with other subjects;

- o   field visits to identify the use of mathematics in everyday life;

- o   continuous assessment;

- o   the use of functional mathematics.

Parental Involvement

It is essential that teachers work with parents as partners in an effort to promote numeracy from the early childhood to the upper primary levels. Parents should be encouraged to participate in activities which can support the development of their children's numeracy skills and hence their ability to perform in mathematics. There are many opportunities for parents and teachers to work cooperatively in enriching children's experience with mathematics. These situations are likely to be the most profitable for two reasons. First, children generally want to please both their parents and their teachers. If they see that mathematics is important to both their parents and their teacher, they will consider it important as well . Second, extending mathematical concepts from the classroom to home will establish the idea that mathematics is not just a school subject, but an everyday subject that makes life more interesting and understandable.

Constant communication between home and school is a significant factor in improving student performance. Schools can explore a variety of activities to engage, equip and empower parents, with the goal being to improve the performance of their children in mathematics. Some strategies/activities which can be used include:

- organizing Mathematics Evenings (within or outside of the PTA setting).

- promoting Family Games Night – and providing parents with mathematically based activities in which they can engage their children.

- providing mathematics classes/workshops/seminars for parents who may lack the knowledge and skills, or may need support in developing the competencies to assist their children in using effective methodologies which are focused on conceptual development. Having provided parents with this practical support, teachers can then encourage them to

assist their children in completing assignments which have been set to reinforce concepts taught during the school day.

- bulletins or newsletters providing practical tips for parents on helping their children make connections between what they are learning in school and the everyday activities in which they are often engaged at home (e.g. baking, farming). These can be designed to meet the needs of parents with children at different grade levels. (See *Putting Mathematics into Family Life* handbook.)

- getting parents involved in decision making processes that impact mathematics issues.

- involving parents in math displays, symposiums and other such activities to promote the importance of mathematics and to provide them with insights on the ways in which they can help their children to succeed in mathematics.

- collaborating with parents to explain how the numeracy development of their children can impact their mathematics performance in later years. In addition, teachers could provide a simplified mathematics benchmark list for parents so that they can know what their children should know and be able to do at the end of each term or school year. (See Chapter 9: *Taxonomy of Numeracy Competencies and Skills.*)

Chapter 3:
Effective Curriculum Delivery

Research has consistently shown that in the teaching of mathematics, practices which are focused on the regurgitation of facts and the ability to perform meaningless computation rather than those focused on problem solving and critical thinking, are usually ineffective and limited in their ability to support meaningful learning. Meaningful learning gives rise to students being able to apply and use related mathematical concepts in contexts outside of the classroom. This is due to the fact that their mathematics learning experiences have been focused on the development of concepts often through the use of concrete objects and resources (manipulatives). Teaching from a conceptual perspective is best done when there is a focus on real life situations and problems, and a greater emphasis on modelling and representation of solutions through diagrams, tables and writing in addition to traditional algebraic processes. Classes which are designed from this perspective often use small group-oriented tasks, where the focus is on thinking strategies and application rather than on an individual display of traditionally emphasised skills, such as speed and recall.

Early Childhood Through to Primary Level Mathematics: Expected Outcomes

Before engaging in any deep discussion of what approach and resultant strategies should be considered and employed in the teaching of mathematics, it is necessary to give some consideration to the question:

> *What mathematics skills do we expect students to display at the end of their primary school encounter with the subject?*

This question is important as its answer will determine to a great extent **what** and – equally as important – **how** mathematics is to be taught to students at the early childhood and primary levels. Generally speaking, the outcomes of a mathematics programme are geared towards addressing

the needs of the learners and society at large. Along this line, research suggests that the teaching of mathematics at the early childhood through to primary levels should perform the following roles.

- It should allow students to develop an understanding of foundation mathematical concepts and ideas and an appreciation for the subject. At the end of their period in primary school, students should not think that mathematics is merely a body of meaningless calculations, rules and formulae that can be used to provide answers to irrelevant, abstract questions. Neither should they see it as simply a 'gate-keeping' subject that plays a practical role in qualifying them for access to post-primary education and eventually a job. Rather, students should come to appreciate that learning mathematics is about developing critical thinking skills and equipping themselves with the tools needed to analyse and describe the environment in which they function.

- It should help students develop the ability to apply appropriate strategies to solving problems in ambiguous situations where the need for mathematics is perhaps only implied. In other words, the aim is for students' numeracy skills to begin to emerge and impact their day to day life and experiences.

- It should support students in developing procedural fluency, that is, the skills needed to carry out procedures flexibly, accurately and efficiently.

- These objectives will result in the primary graduate being able to access higher education, having attained success in understanding and using foundation mathematics concepts. They also are consistent with what has been termed the **three critical prongs** of mathematics proficiency.

  - **Conceptual Understanding**
  - **Computational/Procedural Fluency and**
  - **Problem Solving Skills or Strategic Competence**

Consideration of these three prongs has played a significant role in the development of the National Comprehensive Numeracy Programme.

Teaching Mathematics at the Early Childhood Through to the Primary Level

The National Comprehensive Numeracy Programme is a significant part of the Ministry of Education's plan to address weaknesses in the teaching and learning of mathematics, and as a

result improve the performance of students in the subject. The three prongs of mathematics proficiency identified above act as the foundation for the strategies, goals and aims of the National Comprehensive Numeracy Programme. They are explored in greater depth below.

Conceptual Understanding

The first prong necessary for the development of mathematics proficiency which is emphasised by the National Comprehensive Numeracy Programme is conceptual understanding. Conceptual understanding can be defined as comprehension of foundation mathematics concepts including operations and relationships. Conceptual understanding describes an awareness of mathematical ideas to include the underlying structure and meaning in operations, procedures and processes. For example, in adding 35 and 67, students are likely to arrange the numbers in a column and add corresponding digits resulting in a need to regroup:

$$\begin{array}{r} {}^{1}3\ 5 \\ +\ 6\ 7 \\ \hline 1\ 0\ 2 \end{array}$$

The concept underlying this approach, however, is that of place value which requires that students rename 12 (the result of adding 7 ones and 5 ones) as one group of 10 and 2 ones. Teaching for conceptual understanding is different from the traditional practice of teaching mathematics by focusing lessons on procedures and then exposing children to a number of practice exercises to help them memorise the relevant algorithm or rule. This is known as the algorithmic approach to the teaching of mathematics. The disadvantage to this approach is that it fosters the development of procedural knowledge which manifests itself in a mastery of basic computational skills, algorithms and definitions, but places little to no emphasis on the development of foundation concepts. Procedural knowledge is also limited in that it provides little or no rationale and justification for the rules and procedures which are taught and used. If mastery of mathematics is to be attained, as displayed by a student's ability to apply mathematics concepts and procedures to the solution of everyday problems, then teaching and learning experiences must be focused on developing conceptual understanding.

The Teaching Learning Experience and the Development of Conceptual Understanding

Therefore, what should the teaching learning experience geared towards the development of conceptual understanding look like? The development of conceptual understanding is best facilitated by the use of strategies which are focused on developing concepts rather than on students being able to carry out procedures. Several strategies can be employed to attain this goal. These should include, but are not limited to:

a) **providing students with the opportunity to construct their own knowledge and understanding.** This is best done in a classroom setting which is interactive and which gives students the opportunity to explore with materials and manipulatives. The benefit of focusing mathematics lessons on the development of concepts is that students are given an opportunity to understand the meaning behind established algorithms, and through exploration, to develop their own algorithms while being able to explain and or justify their own responses. Students who have conceptual understanding are also able to independently determine how and what steps to take to solve problems in familiar and unfamiliar contexts. The use of the ten frame as shown below can play a significant role in helping students better understand the relationship between numbers. In the case shown, students with conceptual understanding would be able to appreciate that $8 + 4$ is the same as $10 + 2$. This not only helps them to develop proficiency in addition, but also in understanding why when they add 8 to 4, they 'put down the 2 and carry the 1'.

8 + 4
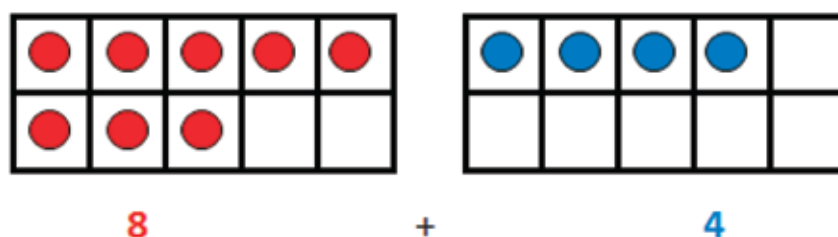
By moving counters from one ten frame to the next, the following is produced:

10 + 2
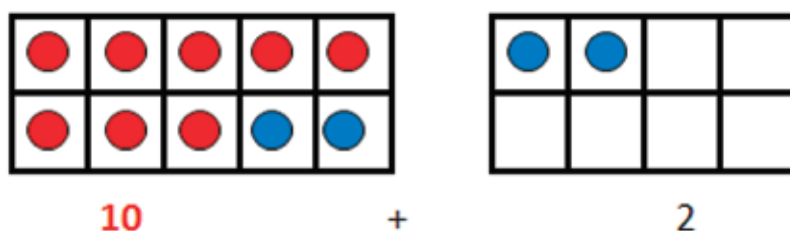
b) **providing opportunities for students to work in groups.** A thinking-based curriculum encourages students to argue and reason as they interact with mathematical ideas. This can be best facilitated by the use of effective grouping of students to promote mathematical discussion and debate. Such groups should be flexible so that they are able to respond to the needs of specific tasks, and small enough to encourage interaction without being too large which could result in a loss of individuality.

c) **providing opportunities for students to explore new and emerging concepts with the use of models guided by effective questioning.** Exploration of the models is best done through the use of free or guided discovery approaches which must be supported by the use of effective questioning techniques. Effective questions draw students' attention to ideas that

are of importance for future development or understanding. Effective questions are those that:

- cause students to give extended responses;
- cause students to investigate or think about their assertions or their responses;
- encourage students to reflect on the choices that they make as they solve problems;
- give students the opportunity to question their convictions.
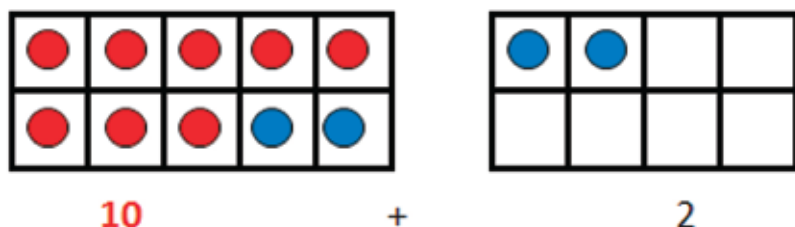
In addition to the strategies highlighted above, it is important that every effort be made to carefully sequence ideas outlined in the curriculum so that students have the requisite foundation on which to build new understanding. If this approach is taken, meaningful learning will take place, minimising the need for students to resort to the mimicking of steps they learn from examining similar problems.

Computational Fluency

The second prong being emphasised under the National Comprehensive Numeracy Strategy is the development of computational fluency.

Computational fluency can be defined as the ability to use number relationships and properties to compute efficiently, particularly when, but not limited to conducting mental calculations. Computational fluency is more than just the ability to recall basic number facts. It is more than being able to exhibit speed in computation developed from extensive and repeated practice. According to Wagner & Davis (2010, p. 40), children with high levels of computational fluency "*have well-understood number meanings… have developed multiple relationships among numbers… recognize the relative magnitudes of numbers and know the effect of operating on numbers.*"

Students who are computationally fluent will display most or all of the following characteristics. The ability to:

a) **invent algorithms and procedures using the properties of numbers to create accurate and meaningful algorithms**. After repeated exposure to addition, for example, a child may appreciate that 48 + 35, though modelled by the teacher, as follows:

$$\begin{array}{r} {}^{1}4\,8 \\ +\ \underline{3\,5} \\ 8\,3 \end{array}$$

can be calculated using any of the following algorithms:

48 + 35 – Method 1	48 + 35 – Method 2	48 + 35 – Method 3
Add the tens: $40 + 30 = 70$ *Add the Ones:* $8 + 5 = 13$ *Add the two partial sums* $70 + 13 = 83$	*Add the tens in the 2nd addend to the 1st addend:* $48 + 30 = 78$ *Add the ones in the 2nd addend to the partial sum* $78 + 5 = 83$	*Make one addend a multiple of 10 by* <u>*adding*</u> *a number:* $48 + 2 = 50$ *Subtract the same number from the second addend:* $35 - 2 = 33$ *Add the two numbers* $50 + 33 = 83$

b) **identify and interpret efficient algorithms through the application of an appreciation for the structure of and relationship between numbers.** Children who have developed this competence, may for example, realise that some numbers are 'compatible' because they are more easily added. As a case in point, children may change 9 + 4 into 10 + 3 (shown below) to make compatible pairs:
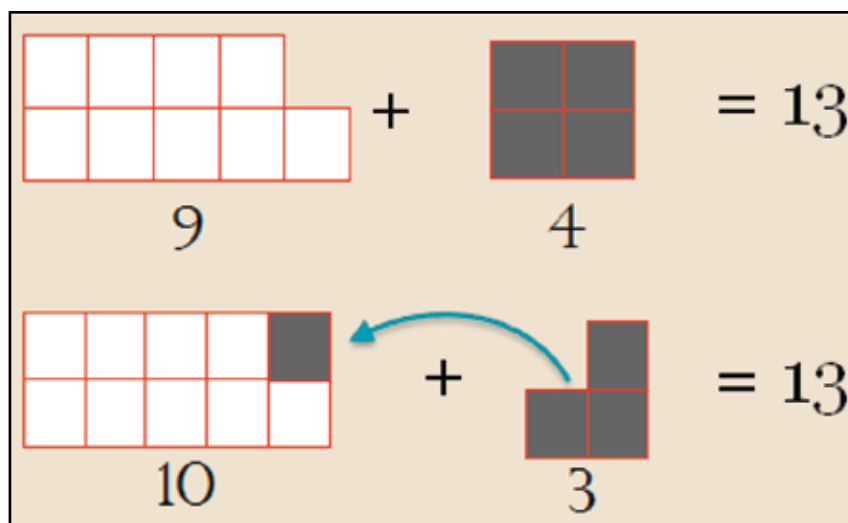
c) **make good estimates**. Computational fluency is also associated with knowledge of the acceptable range within which answers will fall when numbers are operated on. Benchmarks are frequently used to give meaning to numbers and these benchmarks are borne in mind when performingcomputations. For example, if students are able to identify when a fraction is close to zero (such as $\frac{1}{9}$) or to a half (such as $\frac{4}{9}$) or to 1 (such as $\frac{8}{9}$), then they are able to verify the accuracy or assess the reasonableness of their responses and make quick estimates of fraction problems.

d) **write numbers in different ways.** Automaticity is achieved when students are able to quickly state number facts without having to compute. Automaticity in recall of number facts is fundamental if fluency in computation is to be attained. The development of automaticity, however, does not have to rely on constant and intense practice and memorisation of important number facts. Rather, an understanding of the number system and how single- and/or multi-digit numbers can be taken apart and put back together in different ways is helpful in achieving this goal. In evaluating 38×7, for example, one may choose to see 38 as $40 - 2$ and therefore 38×7 becomes:

$$
\begin{array}{r} 40 \\ \times\ 7 \\ \hline 280 \end{array}
\qquad
\begin{array}{r} 2 \\ \times\ 7 \\ \hline 14 \end{array}
\qquad
\begin{array}{r} 280 \\ -\ 14 \\ \hline 266 \end{array}
$$

e) **apply algorithms efficiently.** This is the aspect of computational fluency that is emphasised most often in traditional approaches to teaching mathematics. Efficiency in the application of algorithms describes children's ability to select and use an algorithm without making any mistakes. For children who are computationally fluent, however, application of algorithms is not enough; they must also understand the algorithms and be flexible in their application. They should appreciate the underlying structure of the algorithm and be able to adjust it to respond to changes in the format of the tasks they are attempting to solve.

Developing Computational Fluency

The development of computational fluency must be deliberately addressed in the mathematics classroom. This is best done by paying close attention to :

a) **the student's number sense development**, **particularly since number sense is in itself a pre-requisite for the development of computational fluency**. Number sense describes children's understanding of the magnitude and characteristics of numbers and involves the ability to make estimates and use reference points in computing. By the time a child gets to grade 5, for example, he/she should be able to conclude that all fractions that are close to one have denominators and numerators that are almost equal.

b) **number properties in the exploration of numbers – their relationships and connections**.
Number properties play a significant role in the ability of the student to understand and apply
algebraic thinking such as appreciating that for any two numbers, a and b

$$a + b = b + a$$

c) **the understanding of the properties of numbers that support the development of computational fluency.** This requires that operations and number properties be modelled physically using manipulatives. For example, by modelling odd numbers as follows:
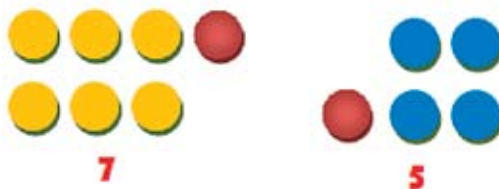
Students can conclude that the sum of two odd numbers always results in an even number:

Students should not be taught rules and procedures without first understanding the concepts and principles to which these rules relate. Understanding and not just memorisation must be the goal of early number operations. For example, having students complete the table below is an excellent way of starting a discussion about multiplying numbers by powers of 10. The activity provides an opportunity for students to identify a relevant pattern and as a result, create their own shortcut rule that is likely to involve placing the appropriate number of zeros at the end of numbers in order to multiply them by powers of 10.

Ten Thousands	Thousands	Hundreds	Tens	Ones	
			4	2	
		4	2	0	× 10
	4	2	0	0	× 100
					× 1,000
					× 10,000

The points above are summarised in the following model:
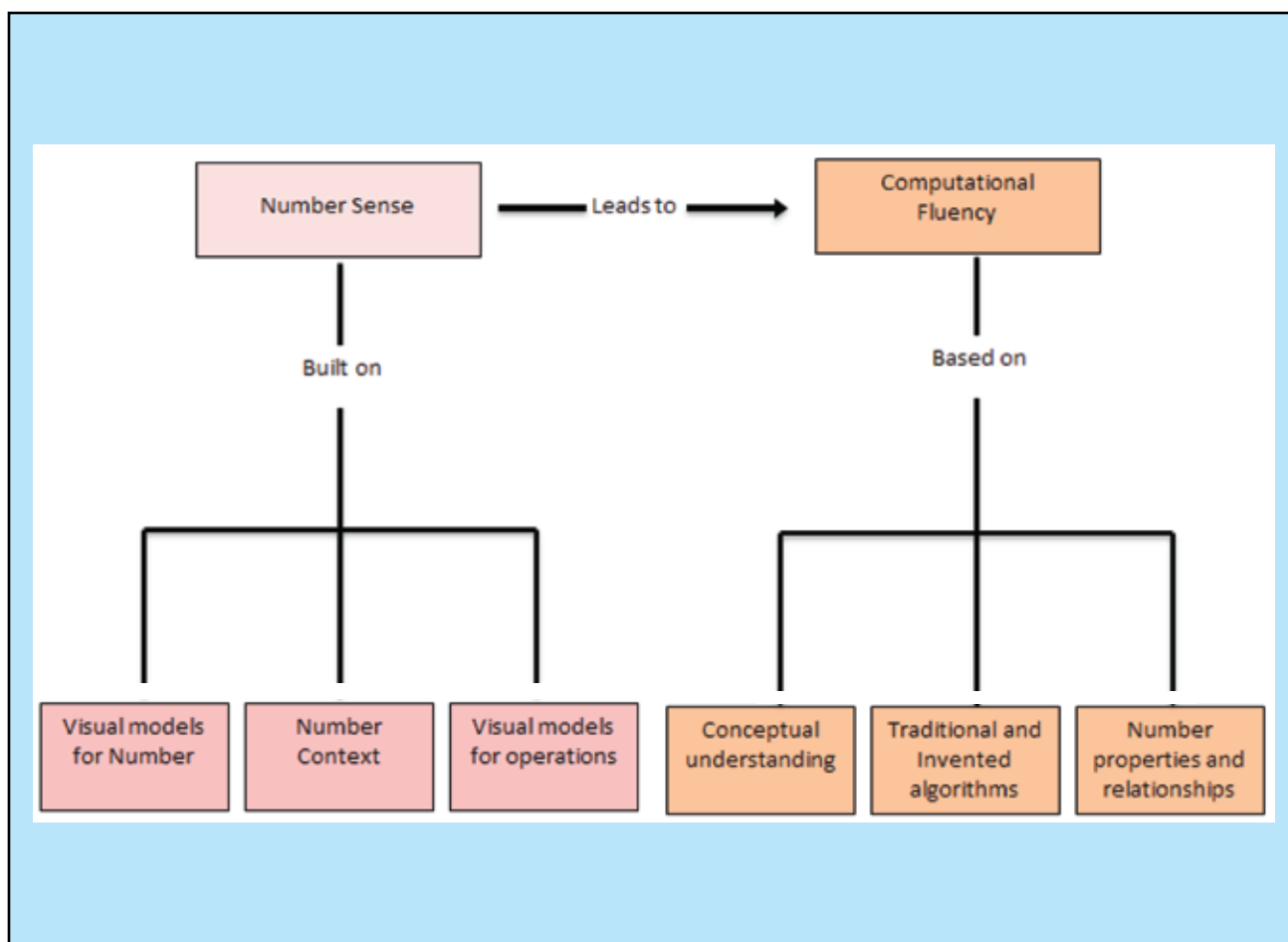
Problem Solving

The third prong in the National Comprehensive Numeracy Strategy is Problem Solving or Strategic Competence.

Students are engaged in problem solving when they are seeking to find solutions to tasks using methods and strategies that they have developed. Students become problem-solvers by being equipped with the skills, strategies and the disposition to attempt any age-appropriate problem, especially those with which they are unfamiliar. Engaging students in problem solving activities is one of the key tools which can be used to support them in developing critical thinking skills. It is important however that teachers recognise that simply making students solve problems is not a guarantee that they will become true problem-solvers. Students, for example, may be coached into solving problems by being shown many solutions to similar problems; however, these students may still be unable to solve problems independently.

Developing Problem Solving Skills

The development of problem solving skills requires consideration of the following principles:

a) **Problem solving is a skill and by its nature cannot be easily 'packaged' and taught to students.** Students develop problem solving skills not just by being exposed to many problems, but also by reflecting on their solutions to these problems and developing general principles from them.

b) **Students' ability to choose strategies must be developed in order for them to solve problems.** Volitional strategies refer to students' ability to act independently of the teacher or other sources of guidance (such as the textbooks or previously worked examples of similar problems) and create solution paths when faced with novel problems. Volitional strategies are developed when teachers expose students to problems without first giving them explicit guidance and/or instructions as to how to complete them.

c) **Traditional word problems are not enough if problem solving skills are to be developed**. For example, consider the word problem below:

> **John has 12 mangoes in a bag and Peter has 8 mangoes in another bag. How many mangoes do they have altogether?**

A close inspection reveals that the problem is a mere contextualisation of the task '$12 + 8 = \_\_$' and is not likely to present students with the opportunity to think critically. Consider, however, the problem as modified below:

> **John has ____ mangoes in a bag and Peter has _____ mangoes in another bag. Together they have 20 mangoes. What are some of the possible number of mangoes that each has?**

d) **Problem solving is not a topic to be taught in the curriculum it is a skill that is developed as students are exposed to the various content strands.**

The Three Prong Framework – Summary

The three prong framework is a multi-focused approach to the teaching of mathematics which ensures that the development of **conceptual understanding, computational fluency** and **problem solving skills** is the aim of primary school mathematics. As an approach, the prongs need not be developed in isolation of each other. In fact, meaningful teaching and learning of mathematics is best facilitated when the teacher seeks to engage students in carefully constructed learning experiences which are designed to support the development of all three prongs simultaneously. The following model shows how the three prongs while somewhat independent are mutually reinforcing and co-dependent.

Problem Solving:

- Students learn concepts <u>to</u> solve problems

- Students learn concepts <u>by</u> solving problems

- Computational fluency is developed <u>through</u> problem solving

Computational Fluency:

- Students invent computational procedures when they understand conceptually

- By computing and exploring number patterns, problems can be solved

Conceptual Understanding:

- Conceptual understanding helps students solve problems

- Conceptual understanding helps students develop computation strategies

Organising for the Teaching of Mathematics

Having identified the objectives of teaching mathematics in the early years and at the primary level and noted therefore the significance of the key strands of mathematics proficiency, it is important that the strands become the focus of mathematics teaching and learning within the Jamaican context. This is more likely to be attained if a system is in place to address the following issues related to the teaching of mathematics:

- **Lesson planning**
- **Teacher support and development**
- **Intervention for struggling students (See section on CTI Model, p.16)**
- **Curriculum expansion activities**
- **Target setting (See Chapter 10: School Accountability)**
- **Assessment (See Chapter 4: The Key Numeracy Outcome Approach)**

Lesson Planning

The first matter which must be addressed if we are to ensure that the strands are the focus of the mathematics teaching and learning experience, is lesson planning. This is because effective planning for the teaching of mathematics is ideally done on a lesson-by-lesson basis. Therefore, even if they are written at the start of the week, mathematics lesson plans should be written for individual lessons or session of length between 45 minutes to 1 hour. It is recommended that these daily lesson plans follow a three part format as follows:

1. **Starter**: A short mental or oral activity that introduces students to the main ideas in the lesson or that gets students interested in or ready for learning. The starter can be a game, a brain teaser or even a story and should last no more than 10 minutes. Starters can also be creative activities designed to support students in developing computational fluency.

2. **Main Activity**: This is the section of the lesson in which the main ideas are developed and in which children are engaged in activities designed to support the development of concepts as they construct their own learning. This section should be between 30 and 40 minutes long and should provide students with the opportunity to learn new material by solving problems and applying new ideas

3. **Plenary**: This section is used to summarize the main ideas in the lesson by providing students with the opportunity to reflect on and share what they have learnt.

Teacher Support and Development

Teacher support and development is another area which will need consistent attention if there is to be a successful change in culture so that conceptual understanding, computational fluency and problem solving become the focus and objective of mathematics teaching and learning in the early years and at the primary level. This aspect of change is critical because it can play a significant role in ensuring that students consistently experience high quality instruction. For this to be attained schools will need to develop a community of practitioners who share ideas and support each other. As noted in Chapter 2 , strategic school leadership is a critical aspect of improving numeracy development. As a result, it is the responsibility of the Principal to ensure that systems are in place to monitor and support teachers in planning for and delivering mathematics lessons that employ effective practices. The Principal can achieve this by ensuring that:

- teachers are exposed to the effective practices that are to be employed in teaching mathematics;

- teachers' capacity to plan and deliver lessons is improved through common planning time, attendance at seminars and workshops designed for this purpose. The design of

these seminars and workshops should provide exposure for the teachers to practices and strategies which promote the development of conceptual understanding, computational fluency and problem solving skills;

- systems are in place to review lesson plans submitted and that timely feedback is given;

- teachers are provided with teaching resources or are empowered to make teaching aids and manipulatives when necessary;

- frequent lesson observations are carried out using instruments designed to effectively evaluate mathematics teaching and learning. Teachers should receive meaningful feedback when this is done;

- teachers receive consistent support from mathematics resource persons identified by the school or within the Quality Education Circle (QEC) and working as a part of the school's Curriculum Implementation Team (CIT).

Curriculum Expansion Activities

In addition to the formal classroom experience of mathematics that students have, there is also a need to ensure that students experience mathematics in novel and exciting ways. An expanded curriculum is one that has facilities which allow students to participate in activities that expand on what they learn in classes. Some of these curriculum expansion activities are outlined below:

- Field trips or other educational trips to see how mathematics is used in real life. Trips to factories or other production plants can be used to get students to talk about measurement, make estimates, perform calculations and solve problems;

- Mathematics Career Days that ensure that students are exposed to the various ways the mathematics that they are learning can be employed to solve real problems in the workplace;

- Mathematics Open Day which allows students to display models and other representations of concepts learnt in the math classroom;

- School census which allows students to collect, represent, analyse and report on data;

- School Based Assessment (SBAs) that ensure that students are required to use the mathematics they have learnt in meaningful ways.

Chapter 4:

The Key Numeracy Outcome Approach

The Key Numeracy Outcomes (KNO): This approach to teaching mathematics is based on the principle that instruction in any mathematics lesson should be geared towards developing a sound understanding of the critical ideas that students need to master the subject. To this end, each mathematics lesson should focus on either developing a concept, facilitating meaningful application of a concept or on providing students with sufficient but focused practice of a concept. Each of these is an essential part of learning mathematics and should form a part of students' experience with the subject. The adoption of the KNO approach requires that distinctions be made among knowledge, skills and concepts.

Knowledge: This is the aspect of students' learning that involves internalising facts, conventions, principles and ideas for recall purposes. When students learn that the horizontal axis runs from left to right and that the vertical axis runs from top to bottom, they are adding to their knowledge base. Symbols for operations and mathematical communication (such as $+$, $\times$, $\div$, $<$, %) are also parts of students' knowledge bank. In general, knowledge describes facts that need not be understood – they simply need to be internalised.

Skills: These are the elements of a mathematics lesson that require students to display learning by carrying out a task – such as adding two numbers; solving an equation; interpreting a worded problem or computing the area of a plane shape. Skills are at a higher cognitive level than knowledge is, as the nature of mathematics problems makes it impossible for a skill to be merely recalled and applied in the same way every time it is to be used. Students must learn skills more thoroughly and with greater clarity than they learn knowledge facts. Indeed, skills should be taught with great emphasis being placed on conventions and principles, but with even greater emphasis being placed on concepts.

Concepts: These are the main ideas involved in learning mathematics and upon which future understanding and skills will be built. Concepts are the core, underlying ideas within a lesson that students must understand in order to master the skills associated with the lesson. It is the learning of concepts that is at the heart of learning mathematics since it is students' understanding of concepts that will allow them to solve problems. Concepts give students the reasons for the steps that they perform in solving a problem or in using a skill. When students learn why digits seem to move around the decimal point when they multiply by 100, then they are learning concepts and not just memorising steps.

While the KNO approach encourages students' development in each of these three areas, it emphasises the learning of concepts as this is the basis of all future proficiency in mathematics. To this end, the following elements of the teaching-learning process are crucial aspects of the KNO approach:

- **Planning for instruction**
- **Appropriate teaching strategies**
- **Assessment**

Planning for Instruction

Planning for instruction is an important activity in which every teacher is to be engaged. This activity is even more important when done for the teaching and learning of mathematics. It cannot be overemphasized that a well-planned lesson increases the likelihood that lessons run smoothly, so that students receive quality instruction. Planning ahead affords you the opportunity to ensure that all elements of the lesson are included and that quality time is spent in delivery of the lesson.

Writing the Lesson Plan

A well-designed three-part lesson gives a clear direction and ensures that desired learning outcomes are achieved. Planning for instruction should be thoughtfully done and should seek to answer some key questions. As you plan for the mathematics lesson here are a few things that you should bear in mind.

- What do I know about this content and what do I need to know in order to teach it?
- What are the underlying concepts of this lesson?
- How are the ideas or knowledge connected to each other?
- What do students already know about this concept?
- What are the skills that they will need to develop in order to learn the concept?
- What manipulatives/resources will be used to aid students' understanding of the concepts?

- Which strategies/methods are best suited to effectively deliver the content of this lesson?

- What activities will the students be engaged in?

- How will students represent their learning?

- How will I assess their learning?

During the delivery of the lesson the teacher should also seek to ensure that key concepts, skills and knowledge are evident. There should be clear differentiation of skills, knowledge and concepts. During the main activity the teacher should ensure that the concepts are not confused with knowledge.

A collection of sample lesson plans have been written by the Ministry of Education's Mathematics Team (see *Sample Lesson Plans*). These lessons reflect the three-part lesson, highlight the difference among skills and concepts, and also outline the skills. The lessons use the KNO approach. You may refer to these sample lesson plans for a guide as to how to develop the three-part lesson.

Management of Resources

One aspect of planning is also how one manages the use of the resources that are available to him/her. Resources are not limited to manipulatives, but include the classroom environment. How is the classroom space used to facilitate grouping as well as whole group instruction? Resources also include textbooks and how they are used and whether there is an over reliance on the use of textbooks. Resources also incorporate the use of technology. Other key resources found in any mathematics classroom are manipulatives . These are physical objects that are used as teaching tools to engage students in the hands-on learning of mathematics. They can be used to introduce, practice, or remediate a concept. One has to be fully familiar with the materials available and how these interact at any given grade level and with the subject requirements so as to result in solid instruction. The Internet is also useful, especially in the areas of research.

Sequencing

One area of mathematics instruction that has always been ignored is sequencing of mathematics ideas. It is important that the teacher sequences the ideas and concepts to be learnt according to the grade level of the child. Mathematics understanding and learning will be more effective and meaningful if instruction and practice are explicitly connected in a sequence. Following a sequence is important because it allows one to follow a pathway throughout mathematics learning. When ideas are sequenced they repeatedly emerge in successive levels of mathematics learning.

Appropriate Teaching Strategies

- The KNO approach heavily stresses student-centred, activity-driven lessons in order to achieve key numeracy outcomes through teaching for conceptual understanding. Therefore, while it is recognized that there is sometimes need for expository teaching, it should not be the primary teaching method used.

- The KNO approach, like the constructivist approach, is built around the student and the premise that students should be allowed to form their own understanding of mathematical concepts. The students are viewed as the masters of their own learning and are given the latitude to create their own understanding of concepts. Therefore, students should be actively engaged in meaningful tasks that facilitate the discovery and development of mathematical concepts.

- It should be noted that the KNO approach does not dictate any one teaching strategy, but rather encourages the use of various strategies. The key is choosing whichever strategy will be the most effective for the concept being taught. The chosen strategy will also depend on the purpose of the lesson.

  - Development of a concept: Discovery learning, problem solving, cooperative learning, discussion, experimentation/investigation

  - Meaningful application of a concept: Discovery learning, problem solving, cooperative learning, discussion, experimentation/investigation, lecture/demonstration

  - Practice of a developed concept: Problem solving, cooperative learning, discussion, lecture/demonstration, drill and practice

Discovery Learning

The discovery learning approach is an inquiry-based approach in which students are engaged in tasks, designed to help them to 'discover' on their own, the desired content. In this approach students are allowed to explore a problem and to formulate their own hypothesis. This exploration is often characterized by exploration and manipulation of concrete objects.

Moore (2005) defines discovery learning as:

> *"…intentional learning through supervised problem solving following the scientific method of investigation."*

Needless to say, the process must be carefully planned by the teacher who should supervise the students closely throughout. While not explicitly directing students' work, the teacher may subtly guide students if needs be through questioning.

Problem Solving

This method engages students in problem solving tasks in order to discover or reinforce the desired content.

Experimentation/Investigation

This method is a student-centred method similar to discovery learning. Here students are faced with a problem and through exploration and manipulation test a proposed hypothesis. Students are encouraged to pose and answer questions they have themselves devised in order to arrive at a conclusion.

Cooperative Learning

In this method students are carefully placed in small groups to work on specific tasks. Students may be grouped based on various criteria; however students are most often grouped according to their ability levels. In such a case they may either be grouped with peers of the same ability level or be placed in a group with mixed ability. Each method has its advantages and the nature of the grouping is dependent on the intention of the teacher. For some teachers, grouping students with matched ability facilitates differentiated instruction. On the other hand mixed ability grouping allows students to learn from their peers.

Discussion

As the name suggests, concepts are explored and developed through small group or whole class discussion. Knowledge is built on students' prior knowledge and experiences.

Lecture

This is a teacher-centred method of instruction in which content is delivered through the direct or explicit instruction from the teacher. This process usually takes the form of a monologue occasionally punctuated by limited feedback from the students or the delivery of written notes in some form or the other (chalk/white board, typed notes, multimedia presentation).

Demonstration

As with the lecture method, this is a teacher-centred method of instruction in which content is delivered through the direct instruction from the teacher. This method differs from the lecture

method in one significant way; here, the intent is for students to learn a particular skill which is physically demonstrated by the teacher. This method is often used in conjunction with the lecture method, with very little input/feedback from students.

Drill and Practice

Drill and Practice is a method used to practice an already developed skill or concept. Students are given several similar tasks to complete, allowing them to practise the particular skill or concept.

It should be noted that with the KNO approach, if algorithms are introduced to students they should be given the opportunity to explore them with a view to determine why they work. This can only be accomplished when students have sound understanding of the concept that is supported by the algorithm.

Additionally, the KNO approach heavily advocates the use of appropriate manipulatives as often as possible particularly in the development and application of concepts

Note: **The Role of the Student**

Unlike traditional approaches to teaching mathematics, the KNO approach emphasises a student-centred approach to teaching. Students should be active participants in the development of concepts and are encouraged to explore various ways of approaching problems. The KNO approach allows students to construct their own understanding of mathematical concepts. To achieve this, the teacher should engage students regularly in problem solving activities as these help to develop just such "habits of mind".

Students are engaged in deliberate meaningful tasks designed to help them to discover concepts. Consequently, particularly in the earlier years, as much as possible, without loss of meaning, students should be given concrete objects to manipulate.

Assessment

How to Assess Conceptual Understanding

The domain of mathematics is so rich and varied that it would not be possible to identify an exhaustive list of concepts. It is important for purposes of classroom assessment, however, for any selection of concepts that is offered to represent a sufficient variety and depth to reveal the essentials of mathematics and their relation to the traditional strands.

The following list of mathematical concepts or big ideas meets this requirement:

- Change and growth
- Space and shape
- Quantitative reasoning
- Uncertainty

Assessment should include:

1. The three types of number relationships (i.e. subitizing, more-less and parts-whole relationships – in the early childhood classroom

2. Multiple ways for number representation – at all levels

3. The three modalities (i.e., visual, auditory and kinesthetic) – at all levels

4. Elements for diagnosing common misunderstandings – at all levels

5. Focusing on the learning outcomes – at all levels

6. Catering to each learning goal more than once and the monitoring of progress toward these goals. Daily progress monitoring will assist in driving instruction as well as interventions.

Classroom Assessment

In the broadest sense assessment is concerned with children's progress and achievement. More specifically, classroom assessment may be defined as the process of gathering, recording, interpreting, using and communicating information about a child's progress and achievement during the development of knowledge, concepts, skills and attitudes. Assessment, therefore, involves much more than testing. It is an ongoing process that encompasses many formal and informal activities designed to monitor and improve teaching and learning in all areas of the curriculum. (NCCA, 2004).

There are two ways to use assessment—as a guide for instruction (**formative**) and as proof of knowledge (**summative**). Teachers should be sure to use both types.

Principles for Classroom Assessment

1. The main purpose of classroom assessment is to improve learning.

2. The mathematics is embedded in worthwhile (engaging, educative, authentic) problems that are part of the students' real world.

3. Methods of assessment should be such that they enable students to reveal what they know, rather than what they do not know (Cockcroft, 1982).

4. A balanced assessment plan should include multiple and varied opportunities (formats) for students to display and document their achievements (Wiggins, 1992).

5. Tasks should operationalize all the goals of the curricula (not just the 'lower' ones). Helpful tools to achieve this are performance standards, including indications of the different levels of mathematical thinking (de Lange, 1987).

6. Grading criteria should be public and consistently applied; and should include examples of earlier grading showing exemplary work and work that is less than exemplary.

7. The assessment process, including scoring and grading, should be clear to students.

8. Students should have opportunities to receive genuine feedback on their work.

9. The quality of a task is not defined by its accessibility to objective scoring, reliability, or validity in the traditional sense, but by its authenticity, fairness, and the extent to which it meets the above principles (de Lange, 1987).

Methods for Classroom Assessment

Teachers are encouraged to use a variety of classroom assessment methods to assess students' development in mathematics. When assessment methods are used properly, students are given the opportunity to demonstrate, in ways that suit their styles, what they can do and understand. Briefly outlined below are some of these methods:

1. **Discussion:** In this method the teacher poses a lead question to start the discussion. For example, at the end of the Grade 4 Statistics unit the teacher can ask the students, "How important is statistics in everyday life?" In this way the teacher can now assess to some degree the students' understanding.

2. **Observation:** This is more than the use of sight. It also involves listening to students' ideas and understanding their reasoning. Observation should be focused on demonstrations and behaviours.

Simple Observation Tool (Topic: Perimeter)

Observation Focus	Yes	No
Were students able to measure around their books and desks?		
Did students give examples of the concept?		
Did students explain the relationship of concept to real world?		

3. **Homework:** As the name suggests, this is the school work that is sent home for students to complete. The main purpose of homework is to further reinforce concepts that were taught earlier. A homework assignment may take various forms, namely written work, research, construction of a model, etc.

4. **Written assessment:** This is what is considered the 'pen and paper test'. This type of assessment takes various forms or a combination of the forms. These forms include essay type or long answer items, multiple choice items, matching items, true or false items and short answer items.

5. **Journals:** These are students' own written entries in their notebooks or in a designated folder. Teachers can use students' journals to recognize errors and develop individualized corrective measures. Journals reflect students' understanding and thinking process as well as their expression of mathematical ideas. As much as possible, feedback is to be provided for the material in students' journals as this will indicate what and how well students are learning.

6. **Oral quiz:** This method is characterized by the exchange of questions and answers by students and teacher, where the teacher poses the questions. This is usually short and each student is given an opportunity to respond to a question. It is advised that the question is asked before a student is identified as this will almost ensure that all students will think about the question, since they would not know who will be asked for a response.

Note: One should note that not all assessment involves grading. Grading everything is counter productive. Focus on the learning goals then find the parts of the units that address these learning goals

Chapter 5:

A Closer Look at Conceptual Understanding

Conceptual understanding answers the 'hows and whys' of a concept, where the teaching and learning process facilitates students' engagement in activities and tasks that allow them to reason and communicate their reasoning. Therefore, a mathematics classroom should provide an atmosphere that is non-threatening and supportive and should encourage the verbalisation and justification of thoughts, actions and conclusions. The intention is to emphasise the teaching of mathematics in a meaningful way with the students actively making sense of the concept presented, in contrast to the more traditional approach where rote learning of procedures constituted a major focus of classroom practice. Additionally, students' levels of understanding of a concept depends upon the quality, extent and connectedness of verbal (written and oral), concrete, pictorial and symbolic forms of representation as it allows them to argue and reason as they interact with mathematical ideas. For this and other reasons, an effective approach in teaching mathematics should be based on students' understanding of basic mathematical concepts from the early years so that students may develop confidence and understanding. The following strands will be explored:

Early Numeracy

Number

Many, if not most of the concepts learnt in mathematics, namely Number, in the early childhood years, are linked to children being able to count. Counting includes both conceptualization of a symbol as representative of a quantity, as well as the recitation of a series of numbers. The concept of numbers develops children's ability to count and to use counting to:

- tell how many objects are in a group;
- compare groups;

- tell which has more and which has fewer objects;

- identify the positions of objects in a line-ordinal numbers-first, second, third, last.

Within the classroom, children should be exposed to number concepts by manipulating materials in the learning centres, songs, rhymes, chants, discussions and number themed activities. These experiences foster children's understanding of recognizing and using different ways to represent numerals, comparing groups of objects and learning to recognize the number words and numerals for numbers.


Counting aids students in developing a conceptual understanding of quantities, place value and the four operations; all of which are necessary for more advanced work in Number.

There are eight known principles of counting:

- **Stable Order Principle**: Understanding that the counting sequence stays consistent. It is 1, 2, 3, 4, 5, 6, 7, etc., or 2, 4, 6, 8, etc., not 1, 2, 4, 5, 8.

- **Order Irrelevance Principle**: Understanding that the counting of objects can begin with any object in a set and the total will stay the same. For example, if there is a line-up of five persons, counting does not have to begin at either end, but counting can start anywhere in the group.

  Example:
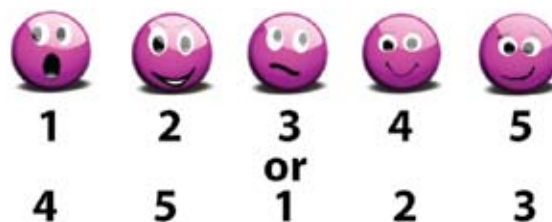
- **Conservation Principle:** Understanding that the count for a set group of objects stays the same no matter whether they are spread out or close together.

  Example:
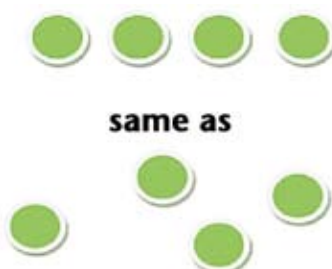
- **Abstraction Principle:** Understanding that the quantity of five large things is the same count as a quantity of five small things. Or the quantity is the same as a mixed group of five small, medium and large things.

Example:
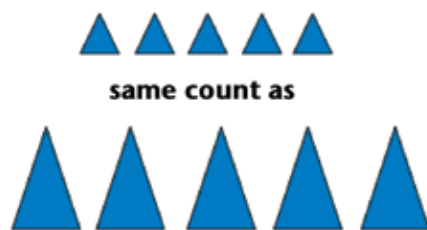
- **One-to-One Correspondence Principle**: Understanding that each object being counted must be given one count and only one count. It is useful in the early stages for children to actually tag each item being counted and to move an item out of the way as it is counted.

- **Cardinality Principle**: Understanding that the last count of a group of objects represents how many are in the group. A child who recounts when asked how many candies are in the set that they just counted, has not understood the cardinality principle.

- **Movement is Magnitude Principle**: Understanding that as you move up the counting sequence, the quantity increases by one, and as you move down or backwards, the quantity decreases by one, (or by whatever number you are counting by, as in skip counting by 10's, the amount goes up by 10 each time).

- **Unitizing Principle**: Understanding that in our base ten system objects are grouped into tens when the count exceeds 9 (and into sets of tens when it exceeds 99), and that this is indicated by a 1 in the tens place of a number.

> **Note:** *See the Jamaican Early Childhood Curriculum Resource Book for Number related activities:*
> - Activity #6: Number Tree, Page 99
> - Activity #1: Letter Hunt (Variation: Number Hunt), Page 149
> - Activity #4: Cards and Counters, Page 153

Measurement

For the conceptual development of measurement at the early childhood stage it is important for children to be actively engaged in measuring activities. Measurement develops children's understanding of the concepts of length, height, weight, capacity, temperature, time and money. Children learn these skills from hands-on experiences with standard and non-standard measurement activities. Standard measurement materials include measuring tapes, scales and timers,

while non-standard measurements include measuring a variety of different objects by using such materials as fudge sticks, seashells, yarn, and crayons. Early Childhood Practitioners can encourage children to learn and compare measurement by providing meaningful activities that include:

- Measuring each child's height once per month and comparing growth from one month to another;

- Allowing children to measure their shoes with strings or paper strips then have them compare their measurements;

- Comparing quantities in the water table using cups and pitchers.

Note: *See the Jamaican Early Childhood Curriculum Resource Book for related Measurement activities:*
- Activity #9: Growing Up, Page 59
- Activity #10: Tone Bottles, Page 59
- Activity #141: Weather Calendar, Page 141
- Activity #11: Rain Gauge, Page 144

Children doing measuring activity (Early Childhood Resource Book, page 55)

Geometry

Young children develop an understanding and awareness that geometry involves shapes and space. Additional geometry involves sorting, serration (order sense), and concepts (directional: right, left, up, down; positions: under, top, bottom, inside, behind; object order: first, second, last, next, before). Exploration of the environment is an important part of the process for learning about the properties of geometry. Children should be provided with activities and

experiences that encourage recognizing and naming specific shapes, drawing shapes, sorting materials by attributes such as size and shape, and placing materials in order by height, length, size, etc. Geometry experiences can be found throughout the daily timetable as Early Childhood Practitioners can encourage children to transition from one activity to another by height, by seating children according to a shape identified on the table, reading books about concepts and patterning.

Note: *See the Jamaican Early Childhood Curriculum Resource Book for related Geometry activities:*
- Activity #12: Play Dough and Toothpick Shapes, Page 122
- Activity #1: Letter Hunt (Variation Shape Hunt), Page 149
- Activity #1: Letter Bingo (Variation Number Bingo), Page 151
- Activity #2: Shape Memory Game, Page 151
- Activity #14: Pass the Bean Bag, Page 163
- Activity #1: Things that Go Together, Page 154
- Activity #4: Matching Colours, Page 155
- Activity #14: Patterned Jewellery, Page 123

Children doing geometry activity (Early childhood Resource Book, page 117)

Statistics and Probability

Statistics provide children with the opportunity to learn how data is categorized and represented in the various forms. Statistical data, known as graphing in early childhood, can be drawn from the children themselves, for example, their favourite pets, food, fruits or games, the number of siblings, month of birth, height, etc. In the onset it is recommended that number sense be developed before statistical exploration. Graphing is a recording device used as a means of

organizing and presenting information through a visual display. As Early Childhood Practitioners make plans to engage their children in statistics and probability the children should be allowed to:

- Sort objects using their observed properties (e.g. size, colour, shape, thickness, texture);

- Create ways for displaying information;

- Estimate and predict based on observations (e.g. weather, number of objects in a jar, taste tests).

Note: *See the Jamaican Early Childhood Curriculum Resource Book for related Geometry activities:*
- Activity #4: Hot and Cold Drinks, Page 52
- Activity #11: Egg Float Experiment, Page 55
- Activity #7: Making Pictographs of Our Taste Tests, Page 68
- Activity #4: Favourite Pet or Animal Graph, Page 76
- Activity #5: Venn Diagram of Jamaican Objects, Page 135
- Activity #10: What can the Wind Blow?, Page 143

Algebra

Algebraic thinking in early childhood is simply problem solving. Problem solving opportunities arise every day in the classroom, for example, when there are not enough pairs of scissors for each child or when a child wants to work in a learning centre that is already at maximum capacity, for example, a fifth child wanting to play in the 'dress up centre' which can accommodate four children only. Early Childhood Practitioners can foster group games that encourage abstract thinking by listening and responding to children appropriately, using mathematical vocabulary and allowing them to solve their problem to the best of their ability.

Note: *See the Jamaican Early Childhood Curriculum Resource Book for related Geometry activities:*
- Activity #8: Who is Missing?, Page 58
- Activity #2: Draw to the Music, Page 157
- Activity #8: Guess the Leader, Page 161
- Activity #11: I See, I See, Page 162

Lower Primary

Making sure that students in the lower primary grades understand the basic concepts of mathematics is an important part of teaching at these grade levels. Without this basic conceptual understanding students will not be able to move forward. It is the underpinning concepts and skills that are developed during these years, thus setting the tone for mathematics success in the future. Early success or lack of it, may factor into the attitude a student has towards mathematics. Therefore it should be ensured that conceptual understanding of any mathematical idea is firmly established in the minds of students at this stage. Outlined below is an example of how conceptual understanding is developed in each strand wherein a primary concept is highlighted.

Number

Place Value

It is critical for students to have a sound understanding of the place value system as it sets the foundation on which other mathematical thinking and processes are built. It is at the heart of teaching children mathematics. Most students know about numbers when they enter the lower primary, however their understanding is limited to counting. Therefore, they should be provided with adequate opportunity to experience place value with respect to composing and decomposing numbers. Also, to further concretize relationships with place value, grouping of items in units, tens and hundreds is essential.

Students need to understand the conceptual framework of how the value of digits within a number changes with regards to their place. Therefore, to begin teaching place value, have students start with an understanding of groups of ten, where they will make a transition from viewing 'ten' as simply the accumulation of 10 ones to seeing it both as 10 ones and 1 ten, which is an important step for students toward understanding the structure of the base ten number system.

Example:

10 ones   =   *1 tens*

Base ten blocks are useful for teaching place value. The units represent 1; the longs represent the regrouping of 10 units for one 10; the flats represent the regrouping of 10 longs for 100.

Units   Longs   Flats
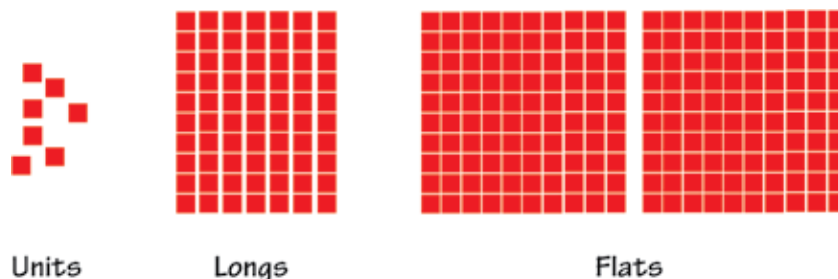
Provide opportunities for students to explore place value concepts using base ten blocks where they are able to see the connection. For example, the following activity can be explored;

- o Students will be given base 10 blocks and a place value chart. The objective is for students to understand that they must group base 10 blocks to minimize the number of blocks that they have.

- o Instruct students to take a number of longs and units, for example, 3 longs and 13 units and regroup the blocks so as to have as few pieces as possible.

3 longs   *13 Units*

- o By exchanging the 10 units for another long then they will record their number (43) in the place value chart

To further reinforce the tens and ones and place value, a hundred chart could be used where students identify numbers based on the place value of each digit.

Hundred Chart

| 1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Digi-cards with numbers from one to ten, multiples of ten and multiples of hundred are also handy manipulatives. Three cards from each group can be put together for students to see face value, value and place value of each digit.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

| 60 | 70 | 80 | 90 | 50 | 40 |

| 100 | 200 | 300 | 400 |

Students can put 400 + 90 + 6 together to get 496, thereby seeing that it's the face value of each digit that forms the number not the value.
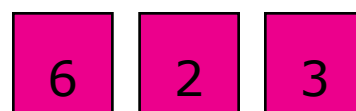
The digi-cards can also be used as reinforcement for students' grasp of place value where the teacher rearranges the cards and students say the new position of each digit.

Example:

| 5 | 6 | 4 | NOW | 6 | 2 | 3 |
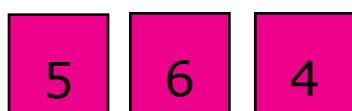
Place value of the 6 is tens   Place value of the 6 is hundreds.

Operations

Students' effectiveness in using operations depends on the counting strategies they have available, and their ability to combine and partition numbers. Students learn the patterns of the basic operations by learning effective counting strategies, working with patterns on number lines and on hundred charts, making pictorial representations and using manipulatives. The operations are related to one another in various ways (e.g., addition and subtraction are inverse operations). Students can explore these relationships to help gain a conceptual understanding of the operations.

Addition and Subtraction

Conceptual understanding of addition and subtraction will form the foundation for more advanced work with these operations as well as form the basis needed for later work in multiplication and division. In adding and subtracting numbers, having students using the counters and the number lines is very important. These models are thinking tools to help them understand what is happening in the problem and thereby keeping track of the numbers and solving the problem.

The relationship between addition and subtraction allow the basic facts to be organised into families. Except for doubles (3+3), each family consists of four related facts.

Example: **4 + 5 = 9   9 – 5 = 4**
                  5 + 4 = 9   9 – 4 = 5

Having students organizing facts into families helps them; once they know the addition facts, they can more easily recall the subtraction facts. It is important to have them work with addition facts from several families together, then focus on the corresponding subtraction facts. One fact family that students should be allowed to explore is the fact family for ten. Sums that equal to 10 are especially important because of the base ten numeration system. After students are skilled at making sums of ten, they are now ready to use this skill to add two numbers greater than ten.

Properties of Addition

Understanding the commutative properties of whole number and the zero property of addition contributes significantly to students understanding of the operations and their ability to master the basic facts. Memorizing the actual words or names of the properties is not critical; however what is important is that students recognize and understand the properties and are able to use them.

- **Identity Property of Addition:** if you add zero to a number, the sum is the same as that given number.

    Example: 5 + 0 = 5 and 0 + 5 = 5

When students are solving addition problems they will encounter a set of zero elements joined to a set with a non zero number of objects. These experiences will help them build an understanding of the addition property of zero.

Example:
Shane had 0 sweets and his friend mark give him 5 sweets.
How many sweets does he have now?

This is a an easy and simple situation that students can solve. Therefore teachers can use problems such as this to assist students to make the generalization that zero added to any number or any number added to zero results in that number.

- **Commutative Property of Addition:** the sum stays the same when the order of the addends is changed.

    Example: 4 + 5 = 5 + 4

A clear understanding of where the commutative property can be applied may help reduce the tendency of some students to always subtract the smaller number from the larger one.

Multiplication

Multiplication requires different thinking than was used in learning addition and subtraction; therefore, teachers need to make sure to address these multiple aspects of multiplication so students can make the appropriate shift in thinking. A major concept hurdle in working with multiplication structures is understanding the group of items as being single while also understanding that a group contains a given number of objects.

Representation

Different representations display different aspects of a particular concept; therefore, a student needs experience with multiple representations to fully understand a concept. The link between visual representations and the multiplication aspect it represents is important for students to grasp at this stage. Students need to actively work with various representations and make connections between them to build conceptual understanding. Below are different representations of multiplication.
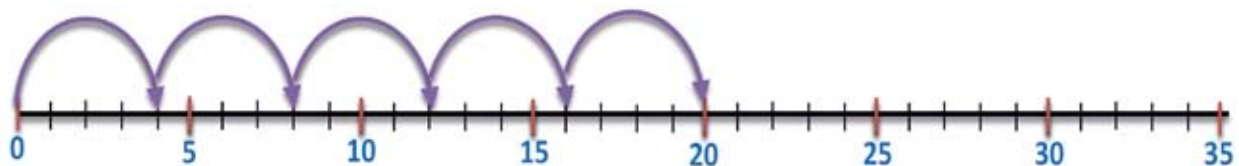
- Representation 1 - Grouping (repeated addition)

4   +   4   +   4   +   4   +   4
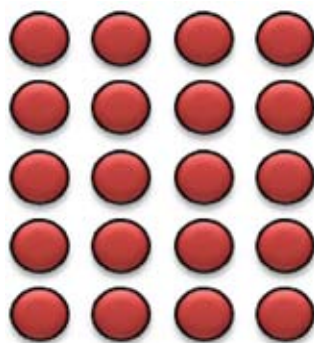
4 x 5 = 20
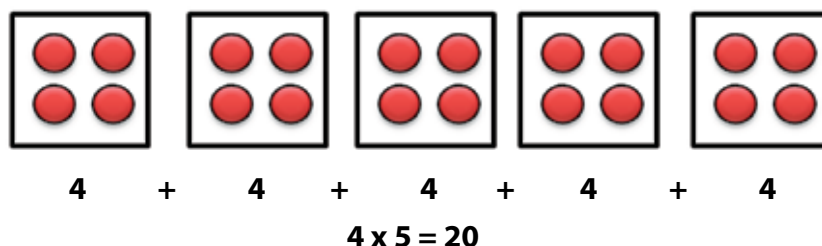
The illustration above represents five groups of four, where the 5 tells how many groups and the 4 tells how many are in each group.

This can also be represented on the number line, for example.

$5 \times 4 = 20$. Five jumps of 4 gets you to 20.

- Representation 2 - Array

Rows multiply by objects in each row equals total number of objects

The illustration above represents 5 rows and 4 columns thus we have $5 \times 4 = 20$

Properties of Multiplication

Like addition and subtraction there are some properties that can assist students' conceptual understanding of multiplication at this stage. These include:

- **Commutative Property of Multiplication:** This allows students to understand that property of multiplication simply means that it does not matter which number is first when you write the problem. The answer is the same.

   Example: 3 x 4 = 4 x

3 x 4   **4 x 3**
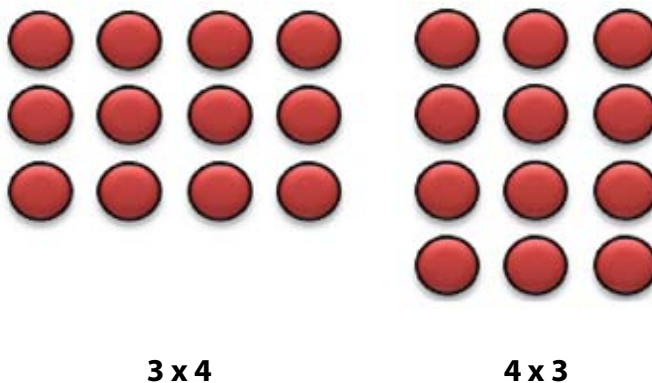
- **The Multiplication Property of One:** This refers to the fact that multiplying by one does not change a number. For example, $4 \times 1 = 4$ or 4 groups of one gives 4. Teachers can have studets develop this concept by having them make model problems that have 1 as one of the factors by forming arrays with one row or one coloum. Such models help students understand the mutliplication property of one.

**1 × 3 = 3**   **3 × 1 = 3**
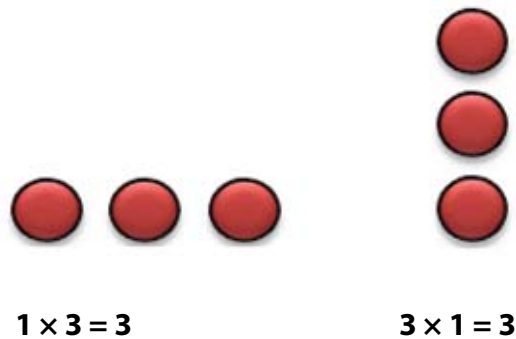
Division

As facts are learned, students need to develop conceptual understanding of what division is and its relationship to multiplication and subtraction. Division should not be taught through rote memorization, but through visual representation, modelling and problem solving that is applicable to the real world. It is important to link the concept of division to sharing objects or things into equal groups as it allows for students to conceptualize division.

Repeated Subtraction

Relate the operation of division to repeated subtraction. As students progress in lower primary they should master subtraction with multiple place values, so you can teach them that they can always use repeated subtraction to solve a division problem. With repeated subtraction, allow them to subtract the smaller number from the bigger one until they get zero, and then count how many times they had to subtract the smaller number. The result is the answer to the problem of the larger number divided by the smaller number.

Example: 20 ÷ 4

You make jumps of four backwards from 20 till you hit 0:
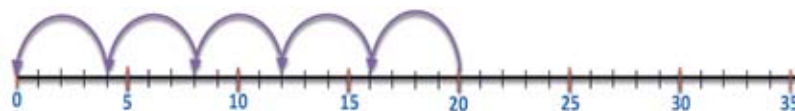
20 ÷ 4 = 5.   20 − 4 − 4 − 4 − 4 − 4 = 0
Five jumps of 4 gets you from 20 to 0.

Partition

Students need to understand the concept of partition when sharing objects into different sets by using division to find the number associated with the partition. This can be modelled using counters, chips or even students. Place a divisible number of counters out and direct students to group the total number of counters evenly into groups or partitions. Students can decide the

number of groups to see if they are evenly divided. For some numbers, like 24, there can be more than one grouping. For other numbers, like 22, there may be only one way to group them. Direct students to construct division sentences based on the outcome of their partition. Students can also be instructed to evenly place 20 students into groups. Specific number of groups can be provided to lead students to find the quantity in each group.

The numbers in division problems have specific names.

- The **dividend** is the number being divided, and this is usually the number with the greatest value.

- The **divisor** is the number the dividend is divided by.

- The **quotient** is the answer to a division problem - the number of times the divisor goes into the dividend.

Fractions

Before students begin to do any form of computation with fractions, it is imperative that they develop an understanding of basic principles surrounding the concept. The first goal in the development of fractions should be to help children construct the idea of fractional parts of the whole. Here, the whole must be represented in the three basic forms: Area or Region, Set and Length models. To develop students' conceptual understanding of fractional ideas, it is necessary to:

- get students to understand sharing equally;

- be proficient in the fraction language (halves, thirds, fourths, etc.);

- demonstrate an understanding of the role of the numerator and the denominator as represented in a fraction;

- allow students to use fraction pieces to model different fractions. These can be used to teach proper fractions, improper fractions, equivalent fractions;

- allow students to use patterns to see how many equal shares make a whole and for them to fully understand what improper fractions are. Allow them also to model fractions as they convert improper fractions to mixed numbers and vice-versa.

Measurement

Conceptual understanding in measurement at this stage is developed with the use of various hands-on activities. This allows students to gain a physical understanding of the different kinds of measurement. At this stage, the aim should be the development of students' understanding of measurement concepts and language through the use of informal measurement units as

they move into more sophisticated strategies and process (standard units). Some of these can be providing opportunities for students to use a variety of non-standard units to measure and discussing how some units are more appropriate than others in particular situations.

Using Non-Standard Units for Measuring Length

In developing sound conceptual understanding in measurement at this level students should be introduce to the concept of measuring using non-standard units of length. This should be done so that students can know how to use a non-standard form of measurement to measure objects around them and how to use estimation to judge the size of items too difficult to measure. For example:

- Length is measured by quantifying the distance from one endpoint of an object to the other endpoint.

- Using non-standard units such as paper clips or fudge sticks can help students partition the length of an object into equal pieces.

  o To measure, allow students to place same-sized units (fudge sticks) end to end to cover the length of an object.

  o In doing this activity students should be given the opportunity to show their ability to use and understand comparative language such as: more/less/the same, most/least; how many more or less; before/after/between; biggest/smallest; half

- By measuring length, you can ensure that students can also develop an understanding that length is conserved if the object changes position.

Comparing and Ordering - Measuring and Estimating Liquid

It is important that students experience activities in which they compare and order attributes as these extend their understanding of the attribute and introduce them to informal measuring processes. Most comparisons however, need to be made indirectly by pouring from one container to another container to see which holds more. Do this to have students realise that two matched amounts of liquid remain the same when one amount is poured into a container of a different shape.

When a comparison between two containers requires the student to find out how much more one container holds, then the concept of capacity is required for students to understand.

- Measuring the area of objects using non-standard or informal units is essential in learning this sequence. Beginning with non-standard, but familiar units such as cups, allows the students to focus on the process of repeatedly using a unit as a measuring device.

- In addition to lots of filling activities using liquids, the students can pack containers with marbles, water or sand.

From the earliest of these experiences, students should be encouraged to estimate. Initially these estimates may be no more than guesses, but estimating involves the students in developing a sense of the size of the unit. As everyday life involves estimating, at least as frequently as finding exact measures, the skill of estimating is important.

When students can measure areas effectively using non-standard units, they are ready to move to the use of standard units. The motivation for moving to this stage, often follows from experiences where the students have used different non-standard units for the same capacity/volume. This allows them to appreciate that consistency in the units used allows for easier and more accurate communication.

The usual sequence used at this level is to introduce the litre as a measurement. Students' measurement experiences must enable them to:

- o develop an understanding of the size of a litre, more than a litre or less than a litre;
- o estimate and measure using litres and millilitres.

The standard units can be made meaningful by looking at the volumes of everyday objects. For example, the litre milk carton, the 2-litre ice-cream container and the 100-millilitre yoghurt container, water bottles, soda bottles, etc.

Geometry

In geometry, the main goal for students at this stage is to achieve a solid practical conceptual understanding of the common two- and three-dimensional shapes and their basic properties, and learn to recognise whole shapes. As students may have had little experience with geometry and geometrical language, they should be given every opportunity to play with objects and talk about their properties and relate basic mathematical shapes to their everyday life.

Shapes

Students should learn about the basic shapes, including triangle, circle, square, rectangle, oval and diamond. They should be able to identify the shapes, draw them and distinguish one from the other. Ensure students build a basic understanding of shapes by allowing them to explore their environment so that they can observe and draw natural and made shapes, and later on identify and name geometric shapes.

To develop conceptual understanding of shapes provide students with opportunities to manipulate, draw, and represent (e.g., on a geo-board) two-dimensional shapes such as circles, squares, rectangles and triangles and facilitate discussion of same. The discussion will encourage students to focus on the attributes of two-dimensional shapes and promote the development of appropriate geometric language. For example, the following questions could be asked:

Why are these rectangles?

Why are these not rectangles? What could be done to these shapes to make them rectangles?

To further capitalize on students' learning of shapes, the teacher can provide them with the opportunity to feel and describe, hence exposing them to the three-dimensional. The teacher can place different shapes in a box (e.g. plastic, wooden or carboard cut-out shapes). Students will be asked, without looking, to take from the box, describe and tell what shape they have found. Students should also be able to locate these shapes found in their environment. Students can also duplicate shapes on a geo-board and then break them into smaller shapes. For an irregular shape, have students find as many triangles, squares, or rectangles within the shape as possible.

Exposure to hands-on activity and real life experiences plays a vital role in students conceptual understanding in geometry. This ensures that as students progress they are engaged in activities

wherein they are able to walk and/or trace open and closed paths, or even create paths of their own and identify open and closed paths from scenarios.

Statistics and Probability

Building conceptual understanding in statistics and probability focuses on getting students to collect, organize and interpret information in practical situations and use simple probability language. Students are required to construct and interpret simple tables and pictographs, using strokes, number pictures or samples to represent data. Students are also expected to read horizontal or vertical bar charts. In probability, students should be given the opportunity to collect, organize and interpret information in practical situations where they describe occurrences as being one of certain, impossible or maybe, and to predict the outcome of experiments and then compare predictions with actual outcomes.

Collection and Organization of Data

For the lower primary, allow students to gather and classify data about attributes, which can be any set of concrete objects that would lend themselves to being sorted and classified in different ways. As students travel throughout the grades this concept enables them to experience the knowledge of collecting and organizing data, and displaying the data using charts and pictograph graphs, including vertical and horizontal bar graphs. Much of the data collected at this stage will be real objects themselves. For example:

- The students can collect shells or a pencil from everyone in the class. Once the data (objects) are collected they can be sorted into categories in preparation for display. It is important that the students are involved in deciding how to sort the objects. Sorting is an excellent way to encourage students to think about important features of data that lead to classifications that make sense.

- Data at this level should also be collected in the form of pictures. For example, the students can understand that pictures of cars are more appropriate than the cars themselves for collection. Once more it is all part of the appropriate thinking of this strand if the students are encouraged to make this decision for themselves.

- Displays at this level follow a simple progression from real object displays to pictographs (pictograms). First, use the objects themselves to form a real objects display.

  o For example, using shoes could involve sorting the shoes into groups: shoes with laces, shoes without laces and sandals.
  o Alternatively, the students can collect and sort pictures of vehicles into a display.

|	
Shoes with laces	
Sandals	
Shoes without laces	

Data Relationship

Data relationship involves presenting data in a concise and visual way that makes it possible to see relationships in the data more easily. Here students are expected to gain a conceptual understanding of reading, describing and interpreting data presented in charts and graphs, including vertical and horizontal bars. Using graphs or charts can make it easier to see relationships in data .

- In preparation for learning to read, interpret and describe graphs, students need to first learn how to make graphs themselves. Bar graphs and tally charts are some of the first ways to group and present data relationships and are especially useful in the lower grades. At this level bar graphs should be made so that each bar consists of countable parts such as square objects, tallies or pictures of objects.

Bar Chart

- In a bar chart the pictures are replaced with vertical straight lines or rectangles. The position of these rectangles indicates what they represent and the height of these rectangles tells how many of that objects there are.
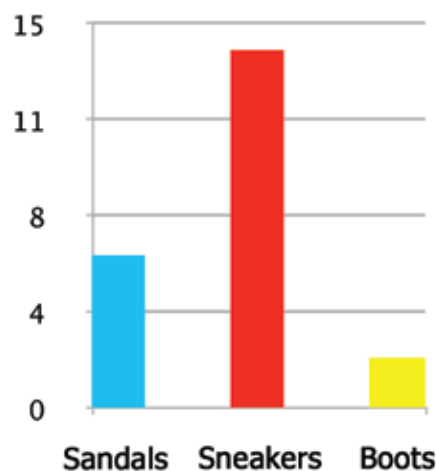
- The example above shows the types of shoes worn in the class on a particular day. There are three types of shoes: sandals, sneakers and boots. The height of the corresponding rectangles shows that there are 6 pairs of sandals, 14 pairs of sneakers and 2 pairs of boots.

Tally Chart

- A tally chart provides a quick method of recording data as events happen. So if the students are counting different coloured cars as they pass the school, a tally chart would be an appropriate means of recording the data. Note that it is usual to put down vertical strokes until there are four. Then the fifth stroke is drawn across the previous four. This process is continued until all the required data has been collected. The advantage of this method of tallying is that it enables the number of objects to be counted quickly and easily at the end.

Red	‖‖ ‖‖ ‖
Yellow	‖‖
White	‖‖ ‖
Black	‖‖ ‖

- In the example above, in the time that cars were recorded, there were 12 red cars, 4 yellow cars, 8 white cars and 7 black ones.

Probability

- At this level students should be given the opportunity to develop the language of probability. The words that are introduced and explored in this unit are **always, perhaps, certain, possible, impossible; will**, **might, won't; will, maybe, never; yes, maybe, no**. These are informal, everyday words that denote chance or probability. By using these words, that have some familiarity for the students, they should start to get a better idea of the overall concept. They should also be introduced to ways of identifying all possible outcomes of an event.

- Students should be given lots of experience with spinners, coins, dice and other random-producing equipment such as drawing a name from a hat or bag.

- The equipment can be used to play games. This should lead to a discussion of fairness of the equipment and to finding the possible outcomes of using it. As they play games, record results and use the results to make predictions, they will find out that with probability they can never know exactly what will happen next, but they get an idea about what to expect.

- Students at this level should be exposed to explore the concept of equally likely events, such as getting a head or tail from the toss of a coin, or the spin of a spinner with two equal sized regions.

- Students can handle simple fractions at this level, and assigning simple probabilities provides them with an interesting and useful application of these numbers. As a result, students should be given the opportunity to see that the probability of getting a head when tossing a coin to be ½.

- Given a spinner that was marked off equally in four colours, ensure that students see that the probability of getting any one of the colours is 1/4, and so on. The explanation would be that there are four equally likely events and that one of them has to happen. Hence, over the long run, you would expect the chance of getting a particular colour is one spin out of three, or 1/4.

Algebra

In developing sound conceptual understanding in algebra teachers must pay keen attention to number sense. The principles of algebra are deeply embedded in number sense. This is true for all levels of algebra.

At the lower primary, in order to be able to complete '*n*' sentences students' understanding of number facts should be properly developed. To develop conceptual understanding, students can be given boxes (with objects) representing different parts of a number sentence.

Example:

After doing this exercise students will understand that even though they are engaged in algebraic activities it requires recalling addition and subtraction concepts previously learnt.

Students should be given ample opportunity to recognize and discuss arithmetic patterns and relationships. For example, in the series 1, 3, 5, 7; students may be asked to determine, with reasonable proof, the pattern of the sequence. Further extension can be facilitated by having the students predict other terms in the series. (The series can also be done in decreasing numerical order based on the level.)

Repeating and creating patterns is an excellent strategy in developing algebra. This can be

done with the use of objects, colours or symbols. In the pictures below students can be asked to recreate the pattern seen. To make it more interesting they can be engaged in creating their own patterns using their choice of display (geometric shapes, drawings, letters, numbers, blocks, etc.) and discussing it with the rest of the class.

Upper Primary

Number

Place Value

At the upper primary level, children focus on large numbers consisting of up to seven digits. The important concepts learnt at the lower primary level – namely, renaming and regrouping – are still relevant at this level, and are expanded upon to assist students to carry out more advanced operations such as multiplication and division and working with decimal numbers.

Multiplication and Division

The most important place value concept underlying mastery of division and multiplication is that numbers can be written in expanded form to show the place and actual values of each digit in a number. This means that the number 96, for example, can be written as 90 + 6. This expanded form of the number is what is at the heart of the 'long division' algorithm. In this algorithm, students focus on digits and not on place value, which usually results in statements such as "I can't". This is illustrated below:

$$4\overline{)268}$$

Traditionally, children start by trying to put '4 into 2', concluding that this is impossible given that 2 is less than 4. They then combine the first two digits to get 26 and now divide that by 4. However, the digit 2 represents a value of 200 from which students can actually get 50 groups of 4. Writing the number in expanded form helps us to see the place value concepts more clearly.

$$\frac{268}{4} = \frac{200 + 60 + 8}{4}$$

$$= 50 + 15 + 2$$

$$= 67$$

This concept can also be used to aid in multiplication. In multiplying 67 by 4, for example, students can be led to develop a general approach by realising that:

$$\begin{array}{c} 6\,7 \\ \times\,4 \\ \hline \end{array} \quad = \quad \begin{array}{c} 6\,0 \\ \times\,4 \\ \hline 240 \end{array} \quad + \quad \begin{array}{c} 7 \\ \times\,4 \\ \hline 28 \end{array}$$

$$= \quad 268$$

Decimal Numbers

Students need to understand that between any two consecutive values, there are 10 equal divisions that can be obtained. Further, each of these 10 equal divisions can be further divided to create even smaller quantities. These small quantities are represented by decimal numbers. An example of this is shown below:

Using this idea, the place value system that students have learnt so far must be extended to include numbers that are smaller than 1 (tenths, hundredths and thousandths); etc. Students' ability to compute with decimal numbers is usually based on their understanding of place value. Otherwise, they will simply compute without an understanding of the process. Dienes blocks or other proportional representations are effective in demonstrating how decimal numbers are made up. In using dienes blocks or other forms of proportional representation, the aim is to get students to understand that 10 of one unit will create a new unit. This is illustrated below:


A whole
1

One tenth
0.1

One hundredth
0.01
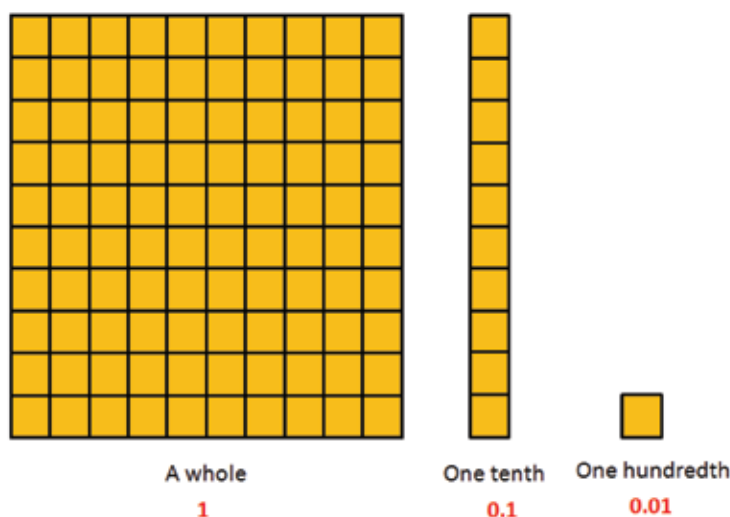
Once students understand this '10 make 1' relationship, then they can start exploring tasks such as the one shown on the following page:

What decimal number can you form using all the pieces below?
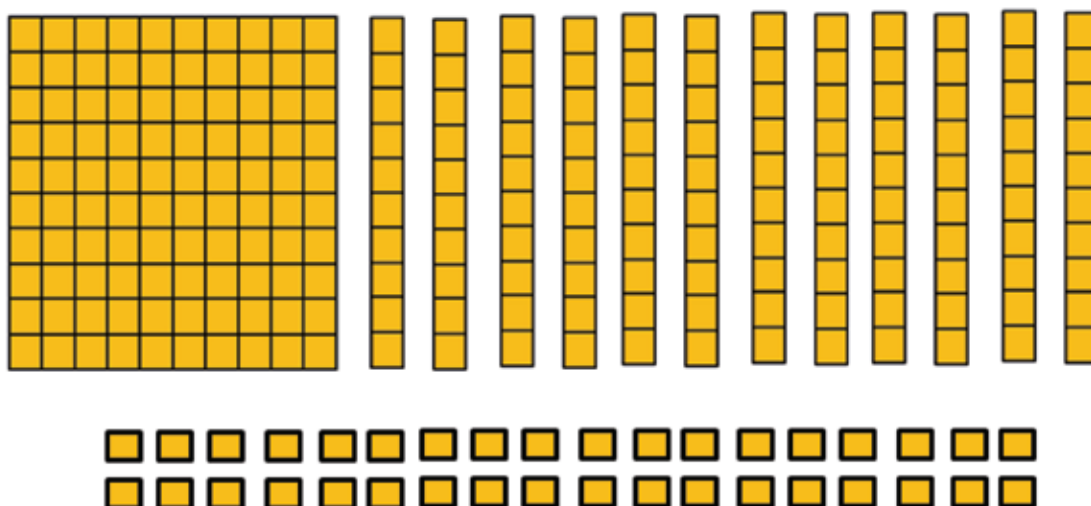
The place value mat is also a useful tool to get students to conceptually understand the ideas associated with decimal numbers. In particular, it can be used to decompose and expand decimal numbers to show the place value position and actual value of each digit. This is modelled below for the number 1.24:

Once students understand the value of digits in numbers, they can then use the place value mat to model how decimal numbers can be added or subtracted. An understanding of the '10 make 1' principle, however, is necessary for them to know why they exchange 10 of the small cubes for 1 long strip or 10 of the strips for one of the flats.
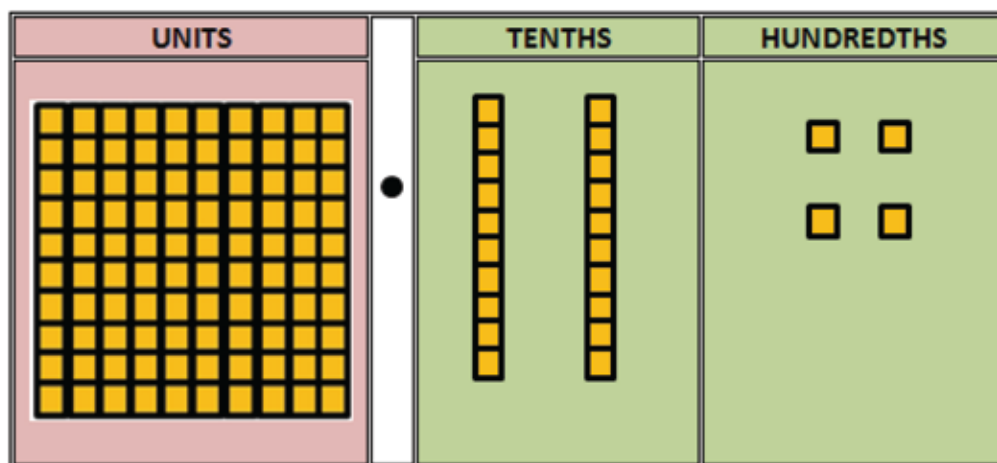

Note: Other ideas in decimals are explored in the *Sample Lesson Plans* handbook – Lessons 10, 11 and 14.

Fractions

At the upper primary level, students' understanding of fractions is heavily dependent on their appreciation for the principle of equivalence. This principle allows students to rewrite a fraction in other forms without changing the value of the fraction. While the approach of the lowest common multiple (LCM) is often used to establish the equivalence of two fractions, this approach should come only after students appreciate the idea of equivalence. This can be done by using fraction tiles to write fractions in different ways. For example, using fraction tiles, students can appreciate that there are many ways to write $\frac{3}{4}$ and $\frac{1}{3}$.
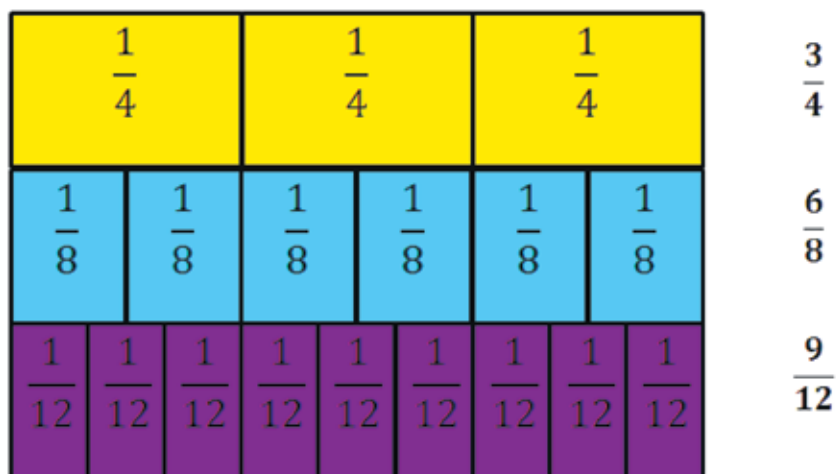
The fraction tiles are used to model that $\frac{3}{4} = \frac{6}{8} = \frac{9}{12}$. This kind of modelling allows students to understand that if the denominator and numerator of a fraction change proportionally, then the resulting fraction is equivalent. Another example, using $\frac{2}{3}$ is shown below.
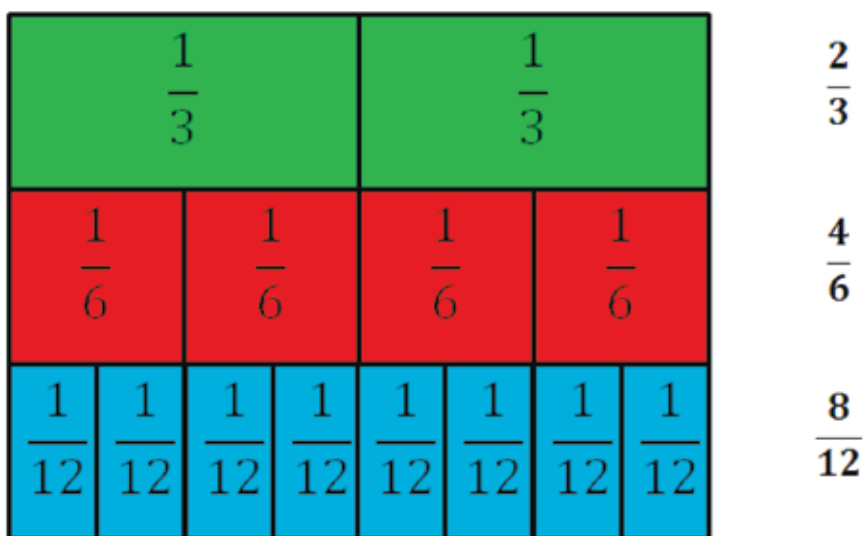
Once students understand how to create equivalent fractions, then having them add fractions with unlike denominators is the next step. Adding $\frac{3}{4}$ and $\frac{2}{3}$, for example, is based on the observation that:

$$\frac{3}{4} + \frac{2}{3} = \frac{8}{12} + \frac{9}{12}$$

Given that the fractions have the same denominator, then, the answer becomes $\frac{17}{12}$ or $1\frac{5}{12}$.

An understanding of fractions also develops from having an idea of the relationship between the numerator, denominator and the value of the fraction. In other words, students must understand what makes a fraction small or large in value. Getting students to appreciate the role of the denominator, in particular, is critical at this level. Specifically, students must understand that unit fractions (such as $\frac{1}{\frac{2,1}{\frac{4,1}{5}}}$) that have large denominators are small in value; this is so because the denominator indicates how many parts the whole has been cut up into. Students can be made to appreciate this point by having them create, pull apart or put together a set of fraction tiles.

Note: Other ideas in fractions are explored in the *Sample Lesson Plans* handbook – Lessons 7, 8, 9, 12 and 13.

Ratios and Proportions

Ratios and proportions describe a multiplicative relationship between quantities. The relationship can be between quantities that are the same, in which case it is called a ratio, or between quantities that are different (rates or proportions). When, for example, one looks at the way in which a child spends his lunch money ($100) on juice and snacks then a ratio is being described. If the child spends $60 on juice and $40 on snack, then the money was spent in the ratio 40 : 60 (snack : juice) or simply 4 : 6 or even simpler as 2 : 3. This means that for every $2 spent on a snack, $3 was spent on juice. In a context where a goalkeeper requires 2 gloves to play a match and hence, 2 goalkeepers require 4 gloves and 3 goalkeepers require 6 gloves, and so on, then a proportion is being described.

Understanding ratios and proportions usually requires that the following be done:

• Use objects that can be grouped easily to introduce students to the idea of ratios. For example, using marbles of various colours, allows students to physically arrange them to see relationships. Consider the task on the following page:

There are 28 marbles below. Arrange them in 4 groups so that the colours are shared equally across the groups.
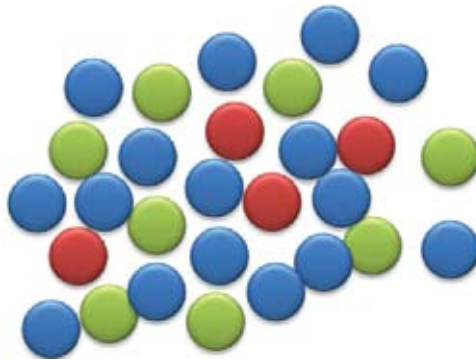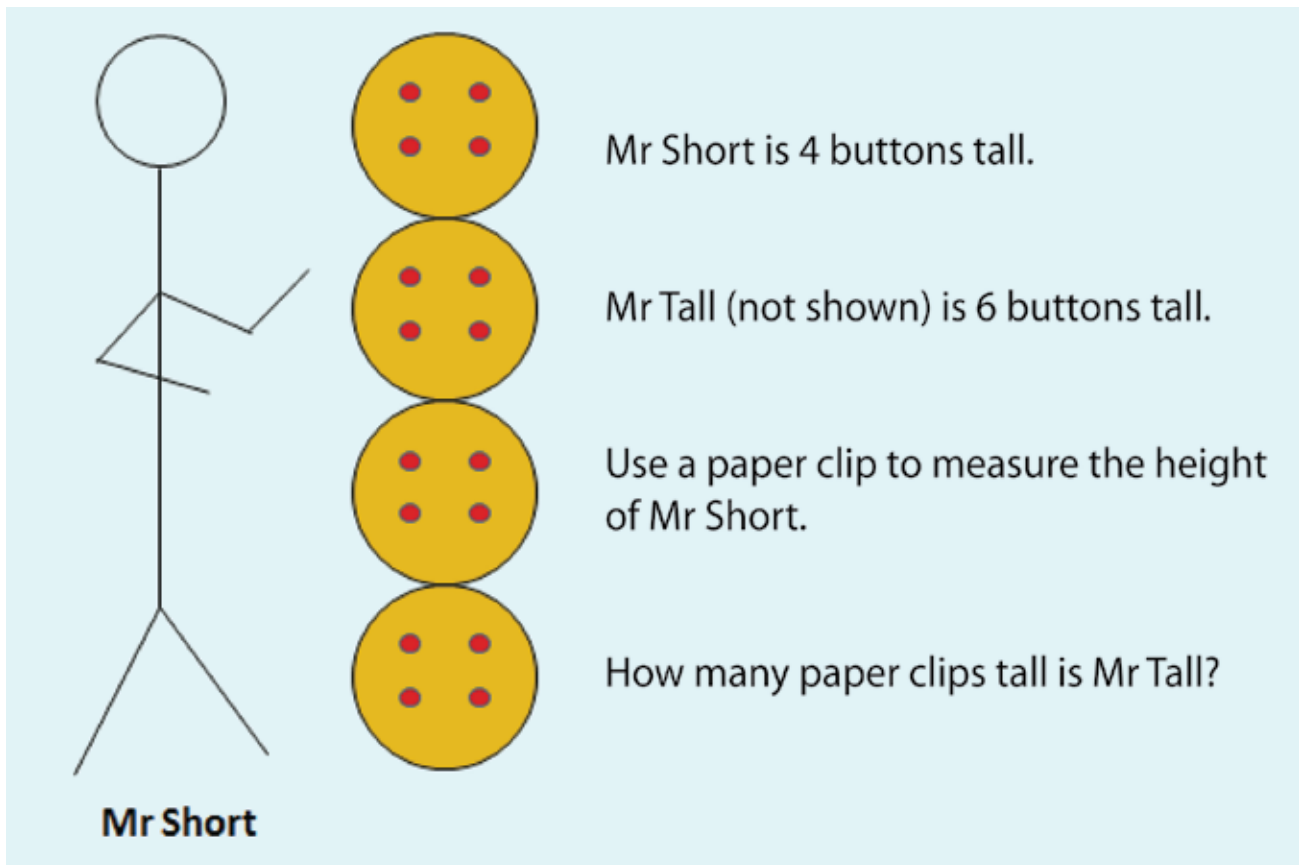
The expected result is shown below:

From discussion of the solution, students can be led to appreciate that for every 1 red marble, there are 2 green marbles and 4 blue marbles. This can be written as 1 : 2 : 4.

- Use many real contexts that allow students to develop ratio and proportional relationships. For example, simply developing male : female relationships using the students in the class is a practical ratio to which students can easily relate. At a more advanced level, scale draw-ings of maps and buildings provide many useful contexts in which ratio relationships can be observed.

- Use multiplicative patterns that are easily observed and described to generate proportional problems and relationships. Observing the relationships between the number of horses and number of legs, for example, and then using that to predict number of legs when there are 30 horses, is a useful exercise that students can engage in as they learn about proportions.

- Have students use known measurements to calculate unknown quantities. The 'Mr Tall and Mr Short' task, for example, is a popular use of this strategy (see page 72).

Mr Short is 4 buttons tall.

Mr Tall (not shown) is 6 buttons tall.

Use a paper clip to measure the height of Mr Short.

How many paper clips tall is Mr Tall?

Mr Short

> **Note:** Ways of introducing students to the ideas of ratio and proportions are explored in greater detail in the *Sample Lesson Plans* handbook – Lessons 15 and 16.

Measurement

Measurement involves the comparison of an attribute of an item or a situation with a unit that has the same attribute (Van De –Wale, 2007). This fundamental concept of measurement that is developed at the lower primary level is further reinforced at the upper primary level.

Measuring Length

Length measurement becomes critically important as students call on the knowledge of this concept to aid understanding of other areas in measurement and geometry. Also, it is important for students to grasp the essential concept of proportional reasoning in the number strand, as at the upper primary level this concept is fundamental knowledge in solving problems relating to scale drawing. Students at the upper primary level should understand the concept of length conservation. Teachers can help students master this concept by having them explore the length of circular objects as well as a straight representation of the same distance. This may be done with the use of strings.

Unit Conversion

At the lower primary level, students are exposed to non-standard units of length measurement and to a lesser extent, standard units of length measurement. Students at the upper primary level will expand on this knowledge by doing their own exploration of the metric units of measurement. For example, students can be given strips of varying lengths, such as a one metre strip along with decimetre and centimetre strips, to determine how many of one makes the other. In addition, they can determine what fraction of a larger unit is a smaller unit. For example, the decimetre is one tenth of the metre. From this activity students can construct their own equations showing the relationships between the metre and the other metric units of length, thereby contextualizing the meanings of the prefixes used.

Measuring Area

Area is the two dimensional space inside a bounded region . Traditionally, students are exposed to the formula for finding the area of some polygon. Most often than not they do not understand how these formulas are derived and why they work.

- For example, with reference to the figure below students should understand that the area of the large square is the sum of the small squares inside the large square. The sum of the squares can be calculated by simply counting the squares or by multiplying the number or squares in a row by the number of squares in a column.
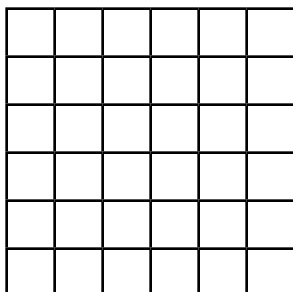
- Using paper folding and tearing techniques, students may explore the idea that every triangle is half of a rectangle. Therefore, the area of a triangle may be calculated by using the formula ½ × base × the perpendicular height.
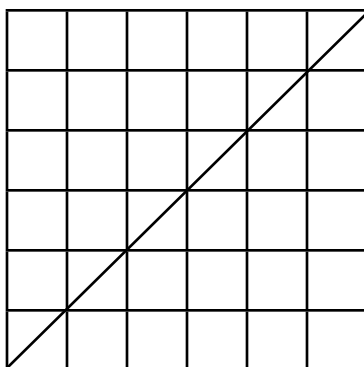
- The area of triangle ABC is equal to half the sum of the total number of small squares inside the large square since the triangle is half the size of the large square.

- The area of a circle can be derived by utilising the concept used to calculate the area of a rectangle/ parallelogram, that is, length x height. For example, using a circle with a radius of seven centimetres (7cm), the circle is then divided into small sectors and arranged to form a rectangle as in the diagram below,
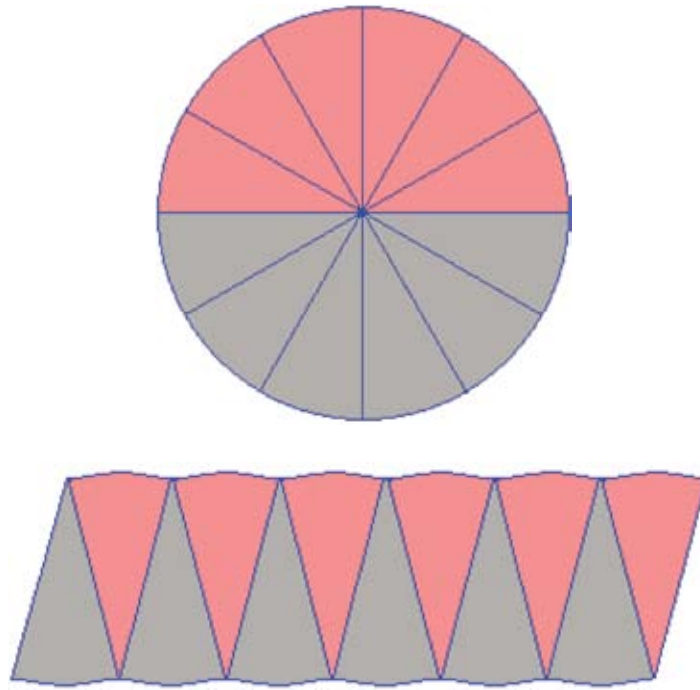
Diagram not done to scale

The length of the parallelogram would be equal to half the circumference of the circle and the radius is equal to the height. The length of the parallelogram then would be equal to π (pi) × r (radius). The area of the parallelogram which is the same as the rectangle is therefore π x r x r.

Measuring Perimeter

Perimeter refers to the distance around a region. At the upper primary level, students continue to explore the concept perimeter and should be engaged in activities that enable them to see the relationships that exist between perimeter and area. For example, students can be provided with 36 square tiles. The task is to see how many rectangles of different dimensions can be made from the 36 square units. Each new rectangle can be recorded by sketching the outline and dimensions on one centimetre grid paper. Students should determine the perimeter for each rectangle. Students will realize that as the rectangle approaches a square shape the perimeter gets smaller.

Students may complete the following table.

Rectangle No.	No. of units of Length	No. of units of Width	No. of units of Perimeter	No. of squared units of Area
1	36	1	74	36
2	18	2	40	36

Geometry

At the upper primary level, students start exploring important concepts in geometry related to two- and three-dimensional shapes and angles. Students learn these concepts primarily through explorations, manipulation, problem solving and reasoned guessing. Teaching methodology that promotes conceptual understanding of the ideas in geometry is one in which children participate in many geometric experiments that allow them to use precise language to describe general principles and make reasoned arguments about properties of plane and solid shapes.

Plane Shapes

Students' experience with plane shapes at this stage builds on their experiential exposure to shapes that they have received so far. At this stage, however, they organise their ideas into categories creating a taxonomy of shapes. This process of creating a taxonomy of shapes involves analysing shapes based on their properties and classifying them in one category or another. The concepts that students need to understand in order to classify shapes are:

- Angle
- Side
- Symmetry
- Congruence

Using geo-boards or dotted paper provides students with many opportunities to explore these concepts. Geo-boards allow students to model and manipulate plane shapes and to observe and make predictions about properties such as symmetry and congruence.

Solid Shapes

Much of the work that students do with solid shapes is developed from their understanding of ideas such as:

- Faces
- Vertices
- Edges

Students learn about these ideas, however, by exploring solid shapes. Indeed, a teaching methodology that promotes creation, exploration and manipulation of solid shapes is best for developing conceptual understanding of solid shapes. Students should not be asked to learn

about solid shapes by imagining or visualising them, even though visualisation activities will follow eventually in an effort to develop students' spatial reasoning abilities. Getting students to manipulate objects and describing them afterwards is crucial if they are to truly understand the main ideas in solid shapes. Students can be allowed to make solid shapes from straws, play-dough or paper. Having students create the nets of various solid shapes before putting them together allows them to truly appreciate that the faces of solid shapes are simply plane shapes. Once students understand the basic appearances of various solid shapes, then they can start classifying them on the basis of their properties such as the number and appearances of their faces.

Statistics and Probability

As students progress to the upper grades conceptual understanding of higher levels of work in statistics and probability still needs developing.

Students should be given the opportunity, preferably in groups, to collect data by conducting surveys or experiments that have to do with their school environment, themselves, or school issues. They should be encouraged to record their observations or their measurements. For example, students can measure their heights to the nearest cm and compile the data for the class, or they could pick an area on the school compound and count the number of plastic bottles found over a four-day period and record their data.

Teachers should give students enough opportunity to compare, through their investigations, different graphical representations of the same data. This will allow them in the future to assess which representation best suites the data with which they are working.

Reading data presented in graphs, tables or on charts is important for students. This will allow them to interpret and draw their conclusions from the data presented. It is important that teachers use the proper questioning to facilitate and guide the process. For example, the graph below shows the amount of kilometres travelled per day by a cyclist. The teacher may start by asking simple questions such as, "What was the distance travelled between Tuesday and Thursday?" The questions will gradually increase in terms of cognitive level.
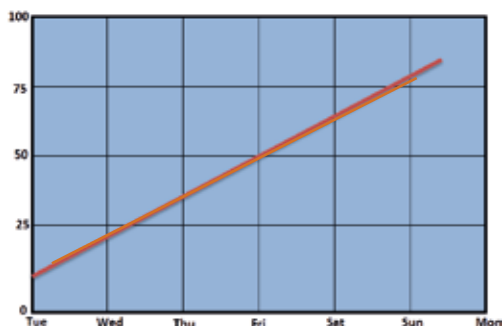
Probability experiments should be conducted and recorded, for example, by rolling a die or tossing coins. The students should be permitted to make their predictions based on the results and then compare their predictions to further experiments.

It is wise to use an approach that involves concrete activities to develop students' conceptual understanding in statistics and probability.

Algebra

One of the central ideas in algebra, on which many others are based, is that of **equivalence**. Two quantities are equivalent if they have the same value, for example, one may say that 2 + 3 is equivalent to 1 + 4, since both of them are equal to 5. Some of the topics for which equivalence is needed in algebra are:

- Equations and n-sentences
- Substitution
- Inequalities
- Order of operations

To show equivalence, the equal sign (=) is used. We therefore, write 2 + 3 = 1 + 4. In other words, the equal sign shows a relationship between two quantities and indicates that values on the left hand side of the equal sign are equivalent to values on the right hand side. It is not uncommon, however, for students learning algebra to use the equal sign to announce that they have completed an arithmetical operation and that the result is what comes immediately after the equal sign. They do not have a relational understanding of the equal sign; rather they hold an operational view, which means that they interpret it as representing the phrase 'the answer is'. As a result of holding this view, students are likely to see the equal sign as a place after which they write the answer to whatever operations came before it or as instruction to 'do something'. As an example, consider the following question:

$$5 + 8 = \boxed{} + 7$$

Many primary aged children are likely to write 13 in the box indicating, erroneously that

$$5 + 8 = 13 + 7$$

The relational understanding of the equal sign is important if the algebraic manipulation that takes place when equations are being solved is to be understood by students. The idea that an expression such as $2(x + 3) - 5$, can be joined with a constant such as 9, by an equal sign to give $2(x + 3) - 5 = 9$, is a manifestation of the relational understanding of the equal sign. It means, as well, that the values on each of the two sides can be changed in a number of ways, as long as

these values change in the same way, thus maintaining balance. Hence an equation remains balanced if the same change occurs on both sides and the unknown can be found by using the 'balancing technique'.

Developing the Idea of Equivalence

Students should be given multiple opportunities to examine equal expressions. This may be explored through a variety of matching or true or false activities which allow students to compare expressions.

Example:

- **23+ 15 = 50 – 12 (true/false)**

- **Which of the following expressions is equal to 12+15?**

    (a) 23+8   **(b) 11+ 25**   **(c) 35 – 8**   **(d) 40-18**

- **Write as many expressions as you can in five minutes that are equal to 11 + 8**

Ideally, these activities are to be given long before students have even started working with equations.

- Develop the concept in early grades to avoid the need to correct later on. As early as grade one students can be allowed to explore the idea of 'balance' around the equal sign. They may be allowed to form equivalent sets using coloured counters. Once the sets are formed they may be asked to write mathematical sentences to describe the sets, using the equal sign to state the relationship between the sets. For example, a student may present the following equivalent sets.
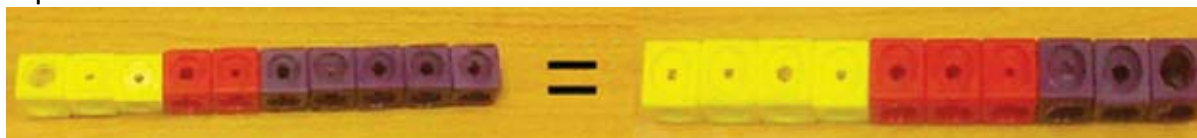

Teachers may choose to utilize beam balances to facilitate students' exploration of this concept:
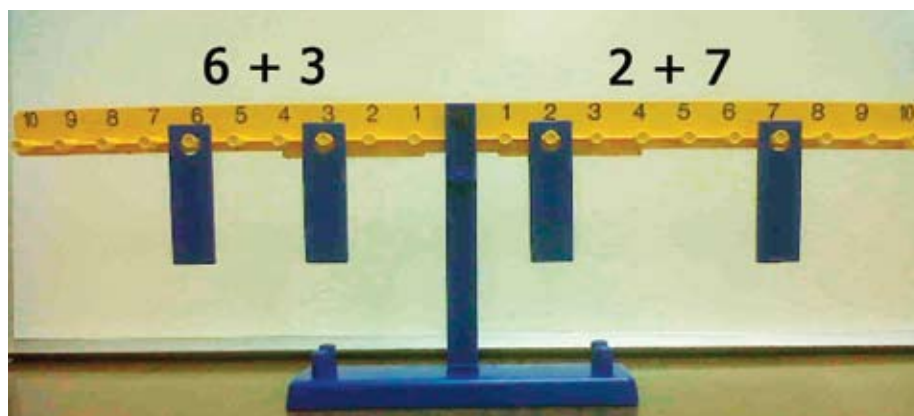
At this level students need not be introduced to the word equation although this is in effect what they are exploring, the focus here is simply to allow students to explore the concept of equality and 'balance'.

Later, students can explore the use of balances to create equal expressions with unknowns.
Example: $2x - 10 \quad \Rightarrow \quad 2x + 5 = 10 + 5$

$$2x = 10 \qquad\qquad 2x + 5 = 10 + 5$$
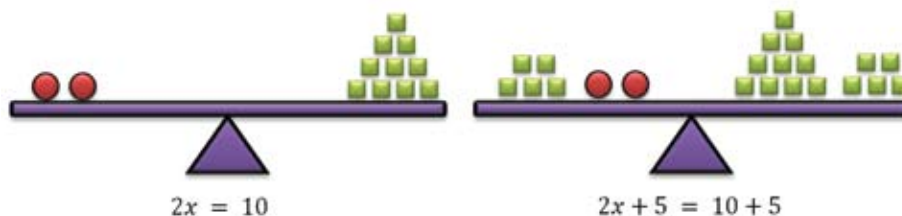
- Students often use the equal sign incorrectly when used to connect steps in an algorithmic solution. For example, if one were finding the sum of the numbers 1, 2, 3, 4, and 5, it is a common error to see students do:

$$1 + 2 = 3 + 3 = 6 + 4 = 10 + 5 = 15.$$

- This way of using the equal sign is incorrect, because each part of the equality has a different value. If interpreted strictly as it says, it implies **3 = 6 = 10 = 15 = 15.**

- A correct solution to the sum would be:
$$1 + 2 = 3, 3 + 3 = 6, 6 + 4 = 10, 10 + 5 = 15.$$

- Ensure, as well, that you do not use the equal sign as a shortcut when choosing variables to represent unknown quantities. For example, in a worded problem where one wants to find Bob's age, do not write:

$$x = \textbf{Bob's age}$$

Rather, write:

$$x \textbf{ represents Bob's age.}$$

- It is also good practice if the left-to-right reading interpretation of the equal sign is challenged from time to time. Children are accustomed to seeing the action taking place before the equal sign and the result coming after:

$$6 + 3 = 9.$$

Challenge this tradition by also writing: **9 = 6 + 3** with some frequency.

Developing Students' Ability to Use Patterns and Functions

- In discussing topics related to patterns and functions, it is important that students appreciate that:
    - identifying a rule that could have been used to create a pattern enables one to generate the pattern indefinitely;
    - when there is a functional relationship between 2 quantities, the value of one quantity determines the corresponding value of the other.
- It is good practice to have students spend a lot of time writing simple relationships using complete sentences and then translating these into algebraic expressions. This helps students make the connection between the complex algebraic form of a relationship and its worded (story) form. For example, consider the table below:

Length (m)	3	4	5	6	7	??	18
Width (m)	5	6	7	8	9	25	??

- Such a table can be a regular part of students' interaction with algebra. Students are expected to examine the table and:

    - Create expressions that describe the relationship between length and width;
    - Use these expressions to determine the missing values.

    Having students examine the table and write down observed relationships in worded form may produce variants of the following:

    - If we add 2 to the length (l), the answer is the width (w);
    - If we subtract 2 from the width, the answer is the length;

    From this, students can be guided into seeing that:

$$w = l + 2$$
$$l = w - 2$$

- Using unifix cubes, straws or paper clips to build towers and patterns is a useful activity to get students thinking about patterns that grow. Students should be able to determine how many cubes, clips or straws would be needed to build the nth tower or to add n stories, etc. Below, for example, are the first 3 occurrences of a tower built using uni-fix cubes:

Getting students to explore these and other towers in the series would allow them to answer questions such as:

a) How many cubes would be in the nth tower?

b) How many stories does the nth tower contain?

c) How many blocks will be in the bottom story in a tower with n stories?

d) How many blocks will be in the bottom story in the nth tower.

Chapter 6:

Developing Computational Fluency

The National Council for Teachers of Mathematics (NCTM) Principles and Standards of School Mathematics (2000) defines computational fluency as having efficient, flexible and accurate methods for computing. Also, to be fluent in mathematics, students should be able to apply mental skills, paper and pencil methods and use technology in computing answers to situations involving numbers. In developing computational fluency at the different levels, a variety of strategies are used. These strategies are ways to think about math facts which lead to more efficient computation because they are based on key number relationships and number properties.

Laying the Foundation in Early Numeracy

Throughout the early childhood years, most children use number sense to solve a problem. Not much happens in terms of computation at the beginning of the early childhood level but as the child nears age five he/she is required to do simple computation. (See Curriculum Guide - The Jamaican Early Childhood Curriculum: pg 124: Intellectual Empowerment, pg 186). By the end of early childhood education, a child should be able to comfortably do the following question without using manipulatives.

"If I give you five sweets, then I give you two more, how many will you have altogether?"

What strategies do early childhood children use to solve this sort of problem? What knowledge (number sense) underlies their strategy choice? Children will be at different levels at the end of the early childhood level; however it is at this level that we begin to lay the foundation for computational fluency. Teachers at this level may use the following activities to help children to get started on a path to compute fluently.

Number sequence from 1 -10

- Allow children to count after you.

- Allow children to take turns counting (saying the number sequence) by themselves.

- Give children practice in saying the sequence serially, with one child saying the number1 and the next child saying the number that comes next, and so on.

- Engage children in activity that has one child start counting, and stopping when the teacher winks or claps. The next child will continue counting where the first child stopped.

- Play "Catch the Teacher". Let children identify counting mistakes that you deliberately make, such as skipping a number or saying a number twice.

- In order to be sure that the results of counting will always tell them how many, no matter what size of the objects or how the units are represented, children need a lot of practice counting sets that are represented in a variety of ways; for example, as objects, dot-set patterns, positions on a path, or points on a dial.

Fluency in identifying set size

- Allow children to count or create sets of different sizes in a variety of contexts; for example, count four versus six objects or walk four versus six steps along a numbered path.

- Encourage children to talk about the results of these procedures in as many ways as they can. For example, they might say, "I'm standing on the number 3 on the path, and I still have a long way to go to the end." These discussions will help children make sense of the procedures they are involved in.

- Encourage children to talk about how sets of different sizes produce differences in magnitude, using the correct terminology for each context. For example, a set of seven toy cars is bigger than a set of five toy cars; seven steps are farther along the path than five steps. These discussions will help children make mathematical or quantitative sense (understanding the relationship between numerals and the quantities they represent) of the procedures they have just completed.

## Counting Strategies for Addition

Counting On

Counting On is a strategy that uses counting to add. These include counting all by sum, counting all from the first addend, counting all from the second addend, counting on from the smaller addend, counting on from the larger addend and just counting one more. These counting strategies can be used with models such as fingers or other physical objects.

Strategy Representation of 2 + 5 = 7

Count All

"1, 2...1, 2, 3, 4, 5...1, 2, 3, 4, 5, 6, 7"

Just One More

"1, 2, 3, 4, 5, 6, 7"
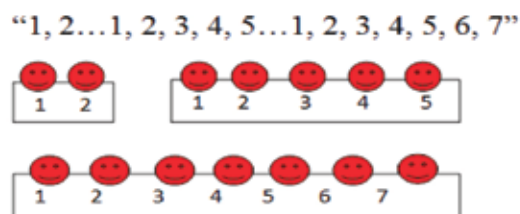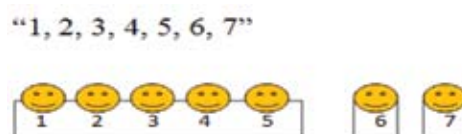
Count on from smaller addend

Student, displays 2 fingers , then 5 fingers ; and says, "7."

Counting on from the larger addend   "5...6, 7"

Counting Subtraction Strategies

Strategy Representation of 7 - 5 = 2

Count Back

Counting Back is a strategy that uses counting backwards to subtract. There are different ways to use counting back.

Counting back from

(7-5):

Jason: "5. . . I counted backwards. . . 7. . . 6,5, 4, 3, 2."

This is by far the most common counting strategy for subtraction. The child has to recite the number names backwards whilst simultaneously keeping a mental (and sometimes physical) tally of the number of counting words said, since this has to match the number being taken away (the subtrahend).

Counting back to

(7 - 5):

Janine: "7... 6, 5, 4, 3. It's two."

This strategy, involves counting down to the subtrahend (here 5). Whilst reciting the number the

child will raise a finger as each number after '7' is said. In this procedure the answer is the number of fingers raised rather than the last number spoken.

Just One Less

Display 7 counters. Take away one each time. Instead of taking away several items, one item is removed each time.

Developing Computational Fluency at Lower Primary

To develop computational fluency at the lower primary level a variety of strategies are used. These strategies are ways to think about mathematical facts which lead to more efficient computation and are based on key number relationships and number properties.
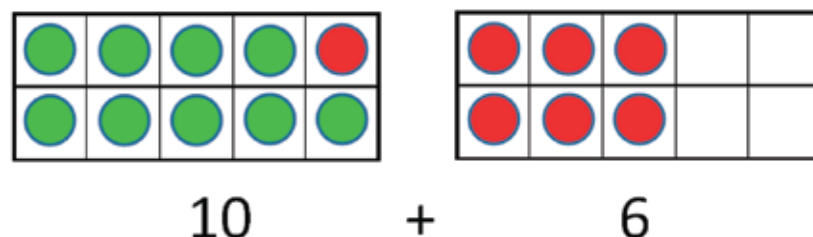
Addition Strategies

Make Ten Strategy

Teachers should work with students so that each student acquires an understanding of several computational strategies and implements them appropriately for the goal of gaining automaticity with basic facts. One strategy to do so is the Make Ten. An example of this is 'see 9, think 10'. Adding 10 to a number is easier than adding 9, students learn to add 10 and subtract 1. Therefore for 9 + 7, the thought process is, 9 + 1 = 10 and 7 − 1 = 6; therefore 10 + 6 = 16.

The sum of 9 and 7 can be illustrated using the ten frame as shown below.

9   +   7
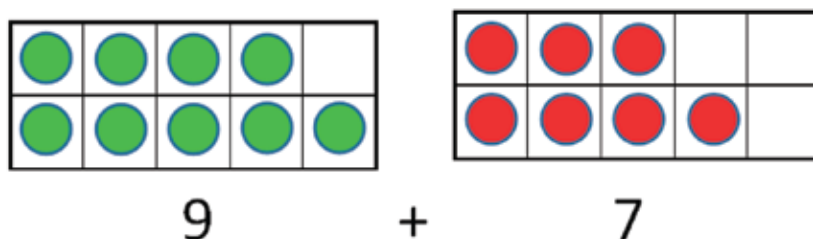
Using the 'make ten strategy', the child will complete one ten frame by taking one counter from the frame with 7 and then add the remaining 6 to make 16. See illustration below.

10   +   6

Rather than simply memorizing facts or resorting to slower processes, students are given ways to think about operations that lead to more fluent computation.

Counting on or Counting Back

Children will use efficient counting strategies, like counting on or counting back. Counting on or counting back are strategies that use a child's knowledge of counting to perform addition or subtraction. For example, facts of ten, students will be able to make groupings showing 5+5, 6+4, 7+3, 8+2. The hundred chart can also be used to show counting on or counting back. For example, given 17-5, the child identifies 17 on the hundred chart and counts back 5 spaces.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Breaking Apart Numbers/Decomposing Numbers

Another strategy that is employed is breaking apart numbers/decomposing numbers. For example, given 7 objects and 6 objects, this can be combined by partitioning the 7 objects into 3 objects and 4 objects, combining the 4 objects with the 6 objects to make one unit of 10 objects, and adding the 3 objects remaining to make 13 objects. This is where children start breaking numbers apart into numbers that are easier to add or subtract (like tens). This way they can use their knowledge of simpler problems to solve more difficult problems. Another way of

decomposing numbers often involves using recall facts , such as Make Ten Facts or Doubles. For example, to solve 8 + 4, a student might write:

$$8 + 4$$
$$2 + 2$$
$$10$$
$$12$$
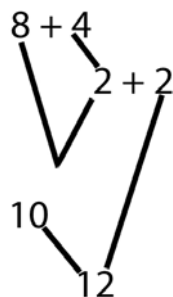
Or, to solve 34 + 38, a student might write:

30 + 30 = 60   **4 + 8 = 12**
60 + 12 = 72

Using Compatible Addends

Compatible numbers are number pairs that go together to make 'friendly' numbers. That is numbers that are easy to work it. You have to choose pairs of addends to make the calculation more manageable. This strategy can be used with two or more addends.

See: 14 + 23 + 16
Think: 14 + 16 + 23

Using Doubles or Near Doubles

Adding a number to itself is a double. This knowledge can be used in aiding addition. For example, when asked to add the numbers 7 and 6. You may double the smaller number and +1 so that 6 + 6 = 12 + 1 = 13. You may also double the larger number and -1, so 7 + 7 = 14 – 1 = 13. Another example might be:

See: 9 + 8 = ____
Think: 9 + 8 = 17 because 8 + 8 = 16, 16 +1 =17 or 9+ 9=18, 18 – 1= 17.

Compensation Strategy

In this stage, you substitute a compatible number for one of the numbers so that you can more easily compute mentally. For example, in doing the calculation 47 + 29 one might think (47 + 30) – 1.

Consecutive Number Strategy

When adding three consecutive numbers, the sum is three times the middle number. For example, 1, 2, 3 is 1+2+3 = 6; which is the same as 3 x 2 equalling 6. 8, 9,10 is 8 + 9+10= 27; which is the same as 3 x 9 equalling 27.

Skip Counting

In developing counting skill of multiplies, skip counting, patterning and the number line can be used. Many times students fail to understand the times tables and often get frustrated. In skip counting students learn to count many things quickly, and by extension, learn their multiplication. When a student skip counts by 3, saying, for example, "3, 6, 9, 12…", then he or she is identifying the multiples of 3 and can deduce from this that 12 can be divided by 3, four times. Skip Counting is counting by a number that is not one. For example, skip counting by 2 can be illustrated on the number line as shown:

The number line is one method used to develop counting skills. Students are able to see movements on the number line. (Skip counting by 2's can be applied as you read a thermometer. Skip counting also helps when learning to read a clock.) It shows how the minute hand progresses when counting by 5.

Count All

Though this strategy was mentioned in the early childhood section, it needs to be strengthened in the early years of lower primary. This basically requires that students count all, which means that if they are adding 8 + 4, they count as follows:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.

Counting On

Counting on means they remember the first number (keep it in their mind) and count up from there, as follows:
(To show how they counted on, they write the numbers they said on their paper, then write some equations for the problem and circle and label their answer.)

Decomposing Number

Another strategy that is employed is breaking apart numbers/decomposing numbers. For example, given 7 objects and 6 objects, this can be combined by partitioning the 7 objects into 3 objects and 4 objects, combining the 4 objects with the 6 objects to make one unit of 10

objects, and then adding the 3 objects remaining to make 13 objects. This is where children start breaking numbers apart into numbers that are easier to add or subtract (like tens). This way they can use problems they know to solve more difficult problems without counting.

Another way of decomposing numbers often involves using a basic fact the student knows by heart, such as Make Ten Facts or Doubles. For example, to solve 8 + 4, a student might write:

8 + 4 = 8 + 2 + 2
    = 10 + 2
    = 12

Or, to solve 34 + 38, a student might write:

30 + 30 = 60   **4 + 8 = 12**
60 + 12 = 72

Adding a number to itself is a double. This knowledge can be used in aiding addition. For example, when asked to add the numbers 7 and 6. You may double the smaller number and +1 so that 6 + 6 = 12 + 1 = 13. You may also double the larger number and -1, so 7 + 7 = 14 − 1 = 13.
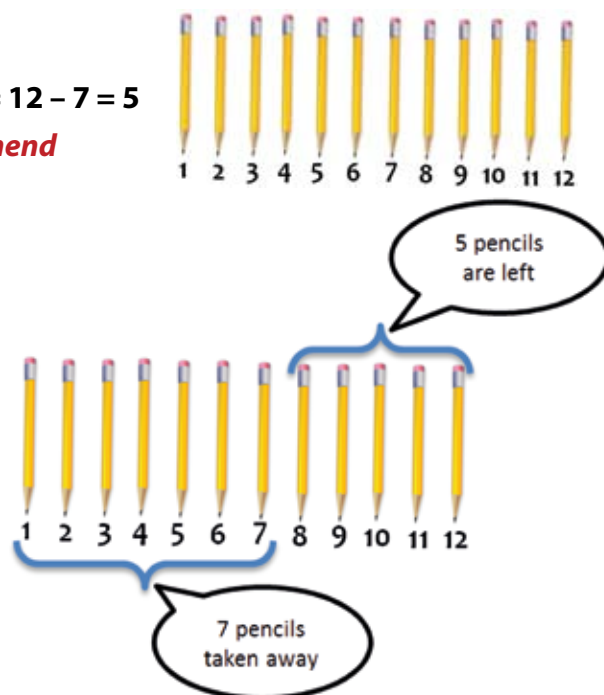
Subtraction Strategies

Draw All and Cross Off

Example:

    There were 12 pencils in a bag. 7 were taken out. How many pencils are left?

Number of pencils left = 12 – 7 = 5

Adding on to the Subtrahend

Example:

If Marco had 63 pigeons and 45 of them flew away how many are left?

Strategy1(add repeatedly with compatible numbers)

$$45 + 5 = 50$$
$$50 + 10 = 60$$
$$60 + 3 = 63$$

Since 5 + 10 + 3 = 18, therefore 63 pigeons – 45 pigeons = 18 pigeons

Strategy 2 (add a number to get close to the original total)

45 + 20 = 65, now 65 is too much, we 2 less to get 63
Hence, 20 – 2 = 18 pigeons

Count Backwards

Counting backwards is the equivalent to counting on when adding: you save the first number in your mind and then count down from there. Hence, 13 – 6 can be illustrated as:

Now 13 – 6 = 7

Decomposing Numbers

Often this involves using a basic fact the student knows by heart, such as Make Ten Facts or Doubles. For example, to solve 8 + 5, a student may write:

$$8 + 5 = 8 + 2 + 3$$
$$= 10 + 3$$
$$= 13$$

$$24 + 27$$
$$20 + 4 \qquad 20 + 7$$
$$1 +$$
$$20 + 20 = 3 \qquad 3 + 7 =$$
$$40 \qquad\qquad 10$$
$$40 + 1 + 10$$
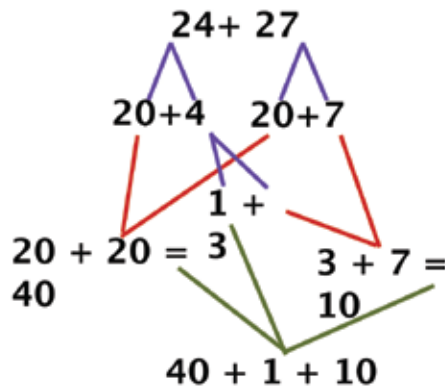
Or, to solve 24 + 27, a student might write:

20 + 20 = 40   **7 + 4 = 11**

40 + 11 = 51

Other examples of this kind of mathematical thinking are:

To solve 25 - 7

I know that 25 – 5 = 20, and 20 – 2 = 18.

OR

I know that 27 – 7 = 20 and 5 is 2 less than 7 so I need to take away 2 more so it will be 18.

Multiplication Tips

The table below shows various strategies that can be used to build computational fluency in multiplication.

Multiplication Tips			
Multiplication by 2	**Multiplication by 3**	**Multiplication by 4**	**Multiplication by 5**
Doubles	**Doubles+1**	**Double Double**	**Clock Facts**
To multiply by 2, double the number.	Double the number, then add it one more time.	Double the other number, then double again.	Think of the minutes on a clock to multiply by 5.
10x2=20	7x3=	9x4=	5x10=50 4x5=20
7x2=14	7x2=14 then, 14+7=21	9x2=18, then 18x2=36	**or**
2x8=16	7x3=21	9x4=36	Multiply by 10 and halve the answer.
2x40=80			5x10, you say 10x10=100, half of 100 is 50. So
			5x10=50 is 100

Multiplication by 6	Multiplication by 7	Multiplication by 8	Multiplication by 9
Clock Facts+1 Set Think of the clock.to multiply by 5 and add one more set. 9x6= 9x5=45 then 45+9=54	**Clock Facts+2 Sets** Think of the clock.to multiply by 5 and add two more sets. 12X7= 12x5=60 then 60+12=72, last 72 + 12= 84	**Double-Double –Double** When one number is 8, double the other number, double the result, and then double again. 7x8= (double one time) 7x2=14 (double a second time) 14x2=28 (double a third time) 28x2=56 7x8=56	**(Decade) 10 -1 Set** Multiply by 10 and subtract 1 set. 6x9= 6 x10 = 60 then 60 – 6 = 54 6x9=54
Multiplication by 10	**Multiplication by 11**	**Multiplication by 12**	**Multiplication by 0**
Decade Facts Multiply x10 (add "0" to end of the number) 7x10=70 16x10=160	**Decade +1 Set** Multiply by 10 and add 1 set 14x11= 14x10=140 then 140+14=154 hence, 14x11=154	**Decade +2 Sets** Multiply by 10 then add 2 sets 13x12 13x10=130 then 130+13=143, last 143+13=156 hence, 13x12=156	**Zero Facts** Any number x0 =0 10x0=0 5x0=0 0x90=0 0x123=0

While these are all number patterns that students should know, teachers should avoid telling students or teaching them as rules. Instead, classroom activities should be designed so that students recognise the patterns and are able to use them effectively in calculations.
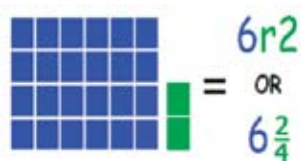
Division Strategies

Grouping

Build it using tiles

26÷4

Take out 26 tiles and put into 4 groups

6 r2 or $6\frac{2}{4}$

Multiples Count bys Strategy

26÷4

Break it Down Strategy

26÷4 can become:

20 ÷ 4 = 5

6 ÷ 4 = 1 with 2 left over

6 r2

Make a Friendly Number Strategy

72÷3

Change 72 to 75

75÷3=25 - 1 =24

There are 3 groups, so add 1 to each group.

Algebra

To develop computational fluency in algebra, students need number skills to determine functional relationships that will allow them to develop generalizations about operations. Students are exposed to patterning with shapes and numbers which are applicable solving problems. Students will be able to efficiently complete n-sentences using a variety of ways. For example, given 6 + n = 10, students should be able to model a number of strategies, some of which could be the use of addition and subtraction patterns, the number line or simple counting

on. In using addition patterns for 10, students will identify which two combinations of numbers make 10. Possible combinations are $0 + 10$, $1 + 9$, $2 + 8$, $3 + 7$, $4 + 6$, and $5 + 5$. Students will be required to match the combinations to the n-sentence problem. Hence, in solving $6 + n = 10$, students identify that 'n' represents '4'.

Another way of developing computational fluency in algebra is by using symbols to represent numerals in mathematical sentences. For example, how many stars remain when five stars are taken from eight stars?

Using an algebraic equation the information can be represented as $8 - 5 = n$, where $n = 3$. Therefore three stars will remain.
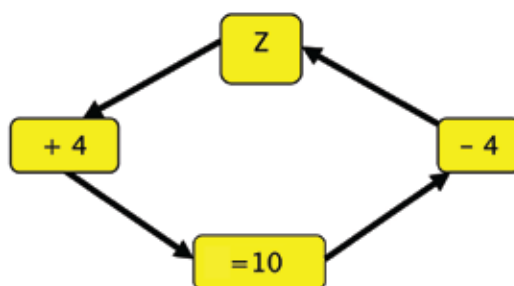

Students should be exposed to the 'reverse process' or 'flow chart' and the balancing methods in solving number sentences. For example:

Using the Cycle Chart
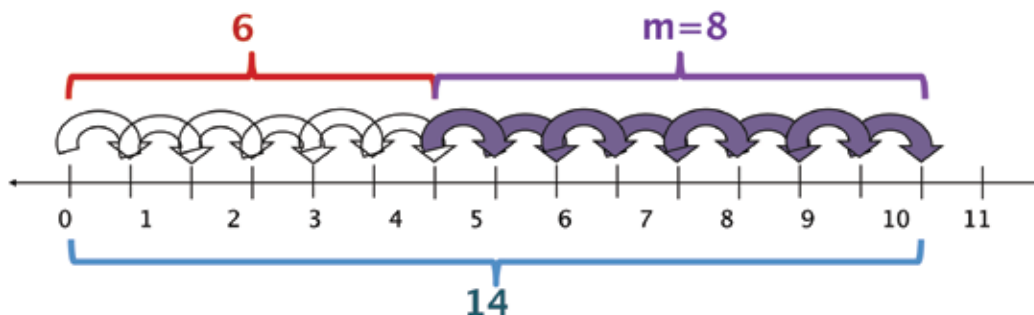$Z + 4 = 10$, student using the cycle chart will be able to use reverse operation in solving the problem.
$Z + 4 = 10$ can be represented by:

Using the balancing method $Z + 4 = 10$ is illustrated as:

$$Z + 4 = 10$$

$$Z + 44 = 10 - 4$$

$$Z + 6$$

Using the number line m = 14 − 6 , what is the value of m?


Developing Computational Fluency at Upper Primary

Addition Strategies

Adding-On Strategy

This strategy involves the keeping of one addend intact, while the other addend is decomposed into friendlier numbers (often according to place value – into ones, tens, hundreds, and so on).

The parts of the second addend are added onto the first addend. For example, given 22 + 57, students might add the first addend to the:
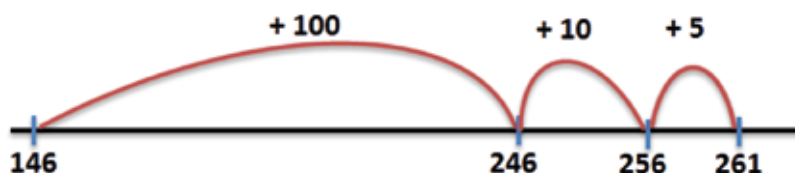
- tens of the second addend (22 + 50 = 72), and then add on the ones of the second addend (72+7=79);
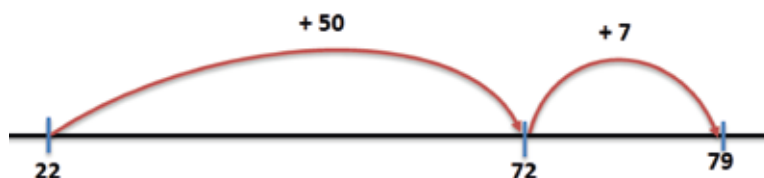

- ones of the second addend (22+7=29), and then add on the tens of the second addend (29 + 50 = 79). The adding-on strategy can be modeled using an open number line.
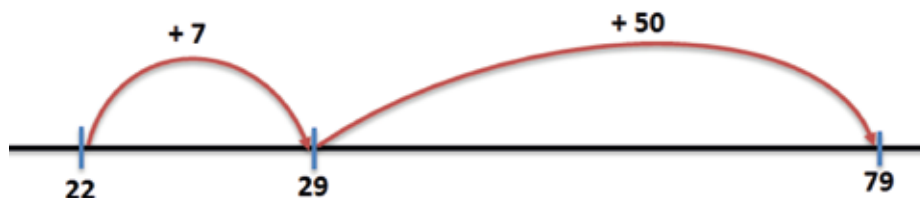
The following example shows 146 + 115. Here, 115 is decomposed into 100, 10, and 5.

The adding-on strategy can also be applied to adding decimal numbers. To add 2.5 and 5.6 students might add 2.5 and 5 first, and then add 7.5 and 0.6. The following number line illustrates the strategy.
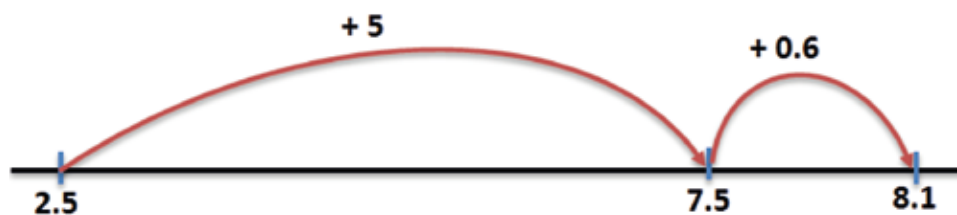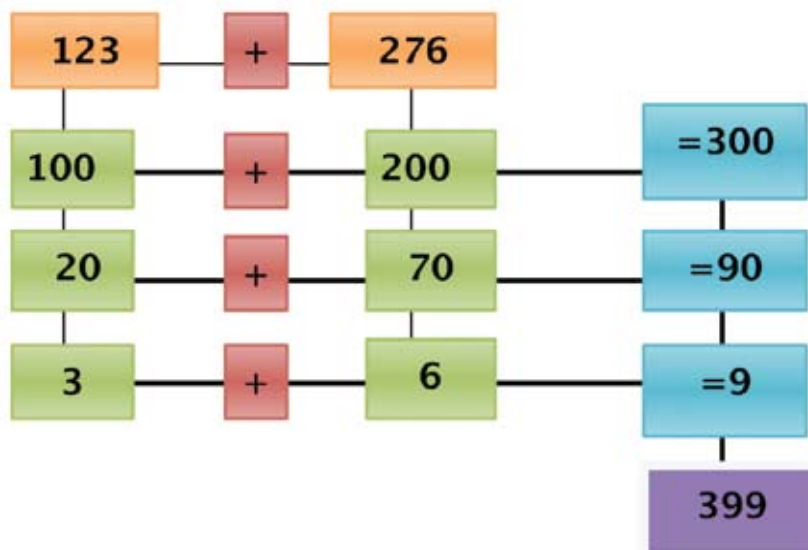

Splitting Strategy

The splitting strategy accentuates the idea that ones are added to ones, tens to tens, hundreds to hundreds, and so on. That is, numbers are decomposed according to place value and then each place-value part is added separately. The partial sums are added to form the total sum.

Example 1:

123 + 276 is composed like this:

100 + 200 + 20 + 70 + 3 + 6 = 300 + 90 + 9 = 399.

Example 2: (see page 97)

Given 3.7 + 6.4 = (3 +6) + (0.7 + 0.4) = 9 + 1.1 = 10.1

The splitting strategy is often used as a mental addition strategy and is less effective for adding whole numbers with four or more digits (and with decimal numbers to hundredths and thousandths), because adding all the partial sums takes time, and students can get frustrated with the amount of adding required. However it is useful at the lower primary levels because it also helps to reinforce the significance of place value in addition.
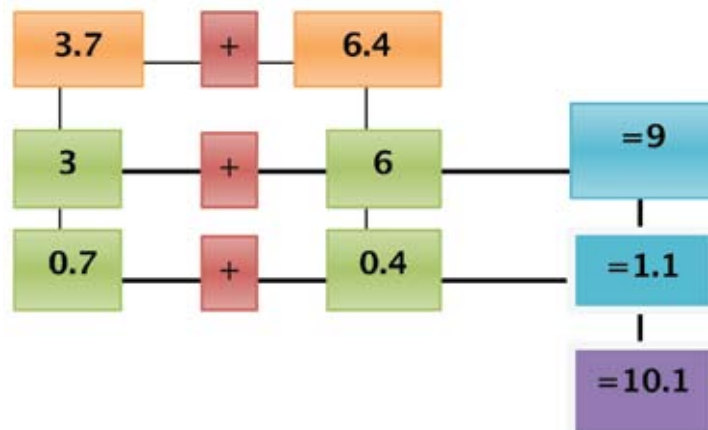
Moving Strategy

This strategy involves 'moving' quantities from one addend to the other to create numbers that are easier to work with. The moving strategy is effective when one addend is close to a friendly number (e.g. a multiple of 10). In the following example, illustrated in the diagram, 392 + 269, 392 is close to 400. By 'moving' 8 from 269 to 392, the addition question can be changed to 400 + 261.
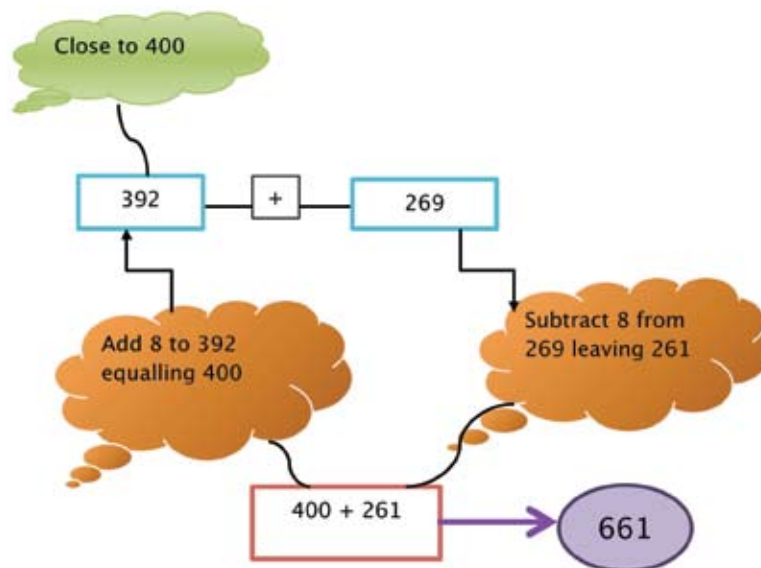
Compensation Strategy

The compensation strategy involves adding more than is needed, and then taking away the extra at the end. This strategy is particularly effective when one addend is close to a friendly number (e.g. a multiple of 10). In the following example, 732 + 180 is solved by adding 732 and 200, and then subtracting the extra 20 (the difference between 200 and 180).

732 + 180

732 + 200 = 932

932 – 20 = 912
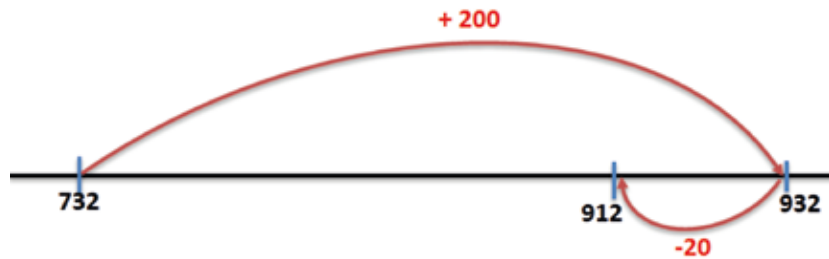
A number line can also be used to model this strategy.

Subtraction Strategies

To develop subtraction strategies there are two interpretations:
1. Subtraction can be thought of as the distance or difference between two given numbers.
2. Subtraction can be thought of as the removal of a quantity from another quantity.

Strategies Based on Interpretation 1

The number line below shows 109 – 83; the difference is (26), which is the space between 109 and 83.
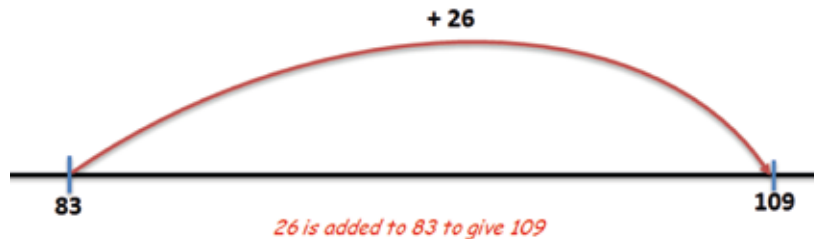
Strategies Based on Interpretation 2

This can be demonstrated using counters. The following illustration shows 15 – 12; the difference is 3, which shows the removal (crossed) counters.
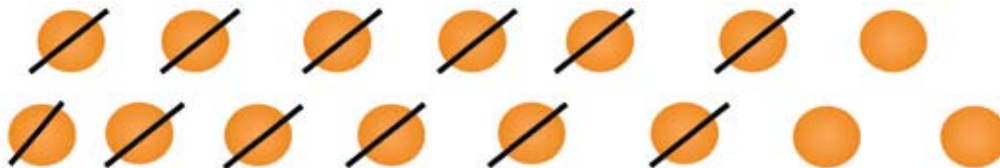
Partial-Subtraction Strategy

With a partial-subtraction strategy, the number being subtracted is decomposed into parts, and each part is subtracted separately. In the following example, 416 is decomposed according to place value (into hundreds, tens, and ones).

$$796 - 416 =$$
$$796 - 400 = 396$$
$$396 - 10 = 386$$
$$386 - 6 = 380$$

The number being subtracted can also be decomposed into parts that result in a multiple of ten, as shown below. Again the number line

$$237 - 145 =$$

$$237 - 145 \longrightarrow 137 + 8$$

$$237 - 137 = 100 \text{ (8 left to be subtracted)}$$

$$100 - 8 = 92$$

Compensation Strategy

The compensation strategy for subtraction involves subtracting more than is required, and then adding back the extra amount. This compensation strategy is effective when the number being subtracted is close to a multiple of 10. In the following example, 341-191 is calculated by subtracting 200 from 341, and then adding back 9 (the difference between 200 and 191).

$$341 - 191$$

$$341 - 200 = 141$$

$$141 + 9 = 150$$

Modeled on the number line, compensation strategies look like big jumps backwards, and then small jumps forward:
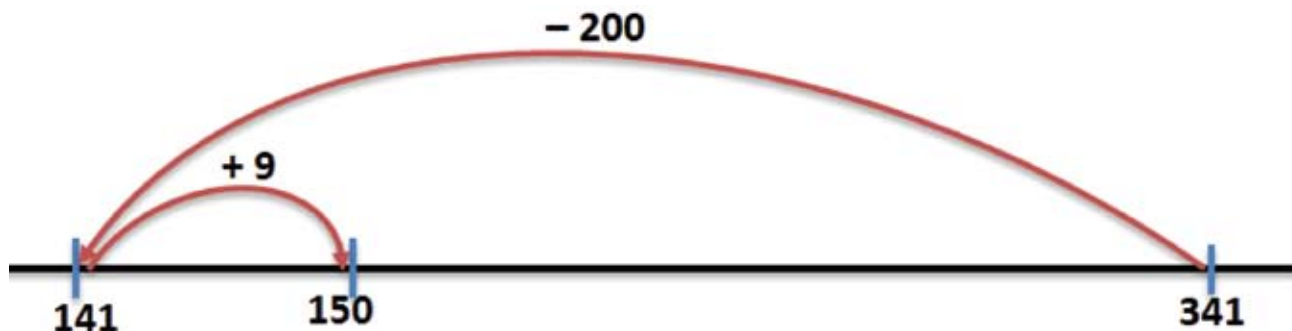
Constant Difference Strategy

Another way of solving subtraction problems mentally is based on the idea of a constant difference. Constant difference refers to the idea that the difference between two numbers does not change after adding or subtracting the same quantity to both numbers. A constant difference strategy usually involves changing the number being subtracted into a multiple of ten. This strategy can be applied using decimals. In the following example, the difference between 480 and 180 is 300. Adding 20 to both numbers does not change the difference – the difference between 500 and 200 is still 300.
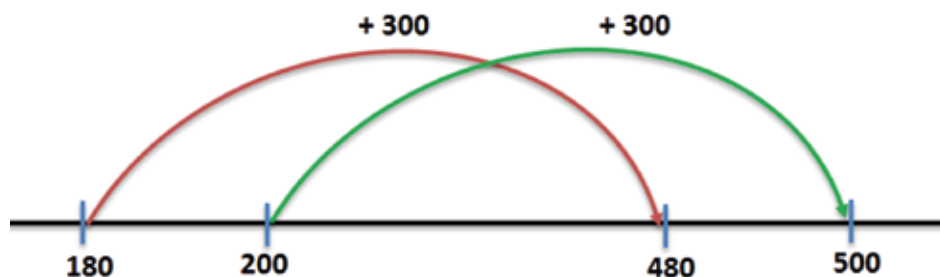
Multiplication Strategies

Developing computational fluency at the grades 4 to 6 level can be fun as students learn their multiplication tables. Below are ways that students can apply to learn their multiplication tables.

Skip Counting

Students can be taught to solve multiplication problems by skip counting, often in conjunction with one-to-one counting and often keeping track of the repeated counts by using materials (for example, fingers) or mental images. For example, a student may solve 4 x 5 by skip counting by fives (5, 10, 15, 20).

Repeated Addition

Multplication problems can be written as addition problems in which one number is added to itself repeatedly. For example, 71 x 4 can be interpreted as 71 + 71 + 71 + 71.

```
      71
+    71
     142
+    71
     213
+    71
     284
```

Partitioning

- **Deriving from known facts:** Learners partition mentally to allow the use of known number facts first, for example, 10 x 13 = 130 so 9 x 13 = 130 − 13 = 117.

- **Place value partitioning:** Learners break numbers into tens, for example, 14 x 5 can be solved as (10 + 4) x 5. 10 x 5 = 50 and 4 x 5 = 20, so the solution is 50 + 20 = 70. Use of place value is also seen in the following example:

This reflects an understanding of base 10.

37 × 4   **276 × 6**

30 × 4 = 120   **200 × 6 = 1200**

 7 × 4 = 28   **70 × 6 = 420**

120 + 28 = 148   **6 × 6 = 36**

                              1,200 + 420 + 36 = 1,656

Compensation Strategies

Looking for strategies to manipulate numbers so the calculations are easier (more familiar).

Double/Half: **27 × 4**

   3 × 18   **30 × 4 = 120**

   (3 × 2) × (18 ÷ 2)   **3 × 4 = 12**

   6 × 9   **120 − 12 = 108**

Distributive Property

This property helps to break down larger numbers into smaller numbers.

Example: 34 × 6

   = (30 + 4) × 6

   = (30 × 6) + (4 × 6)

   = 180 + 24

   = 204

Associative Property

This property uses knowledge of factors.

Example: 12 × 9

   = (2 × 6) × 9

   = 2 × (6 × 9)

   = 2 × 54

   =108

Division Strategies

Divisibility Rules

Students are expected to know and be able to apply the divisibility rules. Teachers can invite students to select 4 digits randomly. Student can then be asked to create as many numbers as they can from the digits that they have identified and then determine whether the numbers are divisible by 2, 3, 4, 5, 6,7, 8, 9 and 10. Calculators can be used for this activity. Prior to the start of the class the teacher would have posted sheets of paper around the room, on which students

would record the numbers that are divisible by the numbers 2 through 10. By careful questioning, students should then be guided to recognise the patterns and develop the divisibility rules.

Divisible by 2	Divisible by 3	Divisible by 4
A number can be divided by 2 if the last digit is even (0, 2, 4, 6, and 8). **Example:** $1\underline{8} \div 2 = 9$ $58\underline{4} \div 2 = 292$	A number is divisible by 3 if the sum of the digits is divisible by 3 **Example:** $21 \div 3 = 7$ Therefore $2 + 1 = 3$ $42 \div 3 = 14$ $4 + 2 = 6$ $162 \div 3 = 54$ $1 + 6 + 2 = 9.$	A number is divisible by 4 if the number made by the last two digits can be divided by 4, or if the number can be halved twice. **Example:** $1\underline{24} \div 4 = 31$ $72 \div 4 = 18$
Divisible by 5	Divisible by 6	Divisible by 7
A number is divisible by 5 if the last digit is a 5 or a 0. **Example:** $25 \div \underline{5} = 5$ • $50 \div 5 = 10$	A number can be divided by 6 if the last digit is **even** and the sum of all the digits is 3, 6 or 9 **Example:** $4\underline{2} \div 6 = 7$ Therefore $4 + 2 = 6$ $114 \div 6 = 19$ $1 + 1 + 4 = 6$	A number is divisible by 7 if when you double the last digit and subtract it from the rest of the number the answer is: • 0, or • divisible by 7 **Example:** $672 \div 7$ 672 (Double 2 to get 4, 67 - 4 = 63, and $63 \div 7 = 9$) Therefore $672 \div 7 = 96$ Also, 705 (Double 5 is 10, 70 - 10 = 60, and $60 \div 7 = 8\ ^4/_7$) Not divisible by 7.

Divisible by 8	Divisible by 9	Divisible by 10
A number is divisible by 8 if the number made by the last three digits will be divisible by 8. **Example:** 176816 ÷ 8 176**816** (816 ÷ 8 = 102) Therefore, 176816 ÷ 8 = 22102	A number is divisible by 9 if the sum of all the digits is divisible by 9. **Example:** 2304 ÷ **9** 2304 (2 + 3 + 0 + 4 = 9) 9 is divisible by 9. Therefore, **2304 ÷ 9 = 256**	A number can be divided by 10 if the last digit is a 0. **Example:** **780 ÷ 10** The number ends in 0. Therefore, **780 ÷ 10 = 78**

Estimation Strategies

Estimation is a practical skill in many real life situations and it is important for students to develop the skill in estimating sums and differences. Estimation skills provide an avenue for students to judge the reasonableness of a calculation performed. When estimating, students should select an appropriate strategy that depends on the context of a given problem and on the numbers involved in the problem. For example, consider the following situation.

David needs to buy movie tickets for $645.25, popcorn for $207.60, and a drink for $110.20. How much money should David bring to the movies?
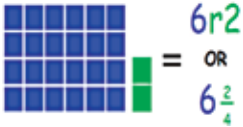
In this situation, students should recognize that an appropriate estimation strategy would involve rounding up each money amount to the closest whole number, so that David will have enough money. The following table lists several estimation strategies for addition and subtraction. It is important to note that the word 'rounding' is used loosely; it does not refer to any set of rules or procedures for rounding numbers.

Strategy	Example
Rounding each number to the nearest multiple of 10, 100, 1000, and so on	786 + 558 is about 800 + 600 = 1400 786 − 558 is about 790 − 560 = 230
Rounding numbers to multiples of ten	786 + 558 is about 800 + 550 = 1450
Rounding one number but not the other	786 − 558 is about 800 − 558 = 242 798 + 558 is about 800 + 558 = 1358
Rounding one number up and the other down **(This strategy is more appropriate for addition than for subtraction.)**	798 + 558 is about 800 + 550 = 1350
Rounding both numbers up or both numbers down. **(This strategy is more appropriate for subtraction than for addition.)**	798 − 558 is about 800 − 600 = 200 798 − 558 is about 700 − 500 = 200
Finding a range	321+ 684 is between 900 (300 + 600) and 1100 (400 + 700) 741 − 249 is between 400 (700 − 300) and 400 (600 − 200)
Using compatible numbers	546 + 125 is about 546 + 124 = 670 648 − 439 is about 648 − 448 = 200

Division Tips

Division tips are indicated in the table which follows.

Division Tips		
Tally Marks and Loops	**Using Tiles**	**Multiples Count bys**
Use Tally Marks and Loops $26 \div 4 = 6 \text{ r2 or } 6\frac{2}{4}$	Build it using tiles $26 \div 4$ Take out 26 tiles and put into 4 groups $6 \text{ r2 or } 6\frac{2}{4}$	$26 \div 4$
The Area Model	**Think in Dollars**	**Break it Down**
First- Build the divisor Second- Fill in the area Third- Build the quotient $26 \div 4 = 6 \text{ r2 or } 6\frac{2}{4}$	Think in $ $100 \div 4$ $\$1.00 \div 4 = \$.25$	$26 \div 4$ can become: $20 \div 4 = 5$ $6 \div 4 = 1$ with 2 left over 6 r2
Make a Friendly Number	**Build it Using Counters**	
$72 \div 3$ Change 72 to 75 $75 \div 3 = 25 - 1 = 24$ There are 3 groups, so add 1 to each group.	$26 \div 2 = 13$ Take out 26 counters and put into 2 groups.	

Chapter 7:

Problem Solving

Introduction and Overview

- One of the most important goals of mathematics education is to improve students' problem solving skills. This can only be realized by ensuring that children are provided with many opportunities to think critically about and engage in solving novel tasks.

- These tasks should allow students to develop and apply problem solving strategies that are the result of their own initiatives because when students merely apply strategies developed and modelled by teachers they do not become independent problem solvers.

- As one of the three prongs of the National Comprehensive Numeracy Programme, problem solving is to be deliberately taught without elevating it to the level of a strand or a topic in mathematics.

- This means that while focusing on problem solving in the mathematics classroom should not be incidental, it should not be separated from other content strands and taught in isolation of other topics or ideas. Problem solving is effectively a tool that students use to access the five content strands – **number, measurement, geometry, algebra and statistics.**

- Given the developmental levels of children, the differing focus of the content at each level and the nature of numeracy associated with various age bands, different aspects of problem solving are emphasised as students advance from early numeracy to upper primary:

  - At the **early numeracy stage**, students' numeracy level is described as **preparatory** as their encounter with mathematics is intended for them to acquire the basic skills – such as counting, sorting and matching – on which all future encounters with the subject is based.

At this preparatory stage, while students may not be solving problems they are being exposed to activities that prepare them to do so at a later stage.

o At the lower primary stage (grades 1 – 3), students' numeracy level is described as being **exploratory** (grade 1) and **developmental** (grades 2 and 3). At these levels, students start exploring content that allows them to describe the real world or to solve problems which require application of real world ideas. Abstract problems that have little or no immediate application to real word scenarios or which students cannot model physically are discouraged in favour of those to which students can relate. Given the emphasis on number ideas in the curriculum at this stage, there should be a commensurate emphasis on problems that require the application of number relationships and computational principles.

o Finally, at the upper primary level, students display either **intermediate** numeracy (grade 4) or '**perceptual'** numeracy (grades 5 and 6). Students are expected to reason abstractedly at these levels and to generalize based on a few cases. The problems that students solve at these stages can be drawn from simulated contexts or may have no real life value or application.

o These points are summarized in the diagram below:

EARLY NUMERACY
a) Children at the **preparatory** Stage of numeracy
b) **Orientation stage** of problem solving through the acquisition of basic skills on which future problem solving activities will be based

LOWER PRIMARY
a) Children at the **exploratory** and **developmental** stages of numeracy and the **experiential** level of problem solving
b) Emphasis on problems that promote physical modelling and real world application of mathematical ideas

UPPER PRIMARY
a) Children at the **intermediate** and **perceptual** stages of numeracy and the **abstract** level of problem solving
b) Involvement in problem solving tasks that allow students to reason abstractedly about simulated problems that may have no immediate application to their daily lives

Early Numeracy

At this stage, students are exposed to tasks, ideas, activities and situations that orientate them to problem solving and which build the basis for effective problem solving in the future. These preparatory problem solving tasks are built on developing an appreciation for the following four major skills:

Patterning and Sequencing

- Patterns describe lists of objects, numbers, sounds, etc., that change in a precise way according to a given repeating rule. Children see patterns in the environment, in numbers and in their actions; their ability to create, identify, describe, repeat, extend and complete patterns will be crucial for problem solving activities as they advance through the curriculum.

- Patterning is an essential skill for students to develop as they need it when they are asked, at a later stage, to solve problems that involve generalisations or those that involve predicting or extrapolating. In addition, patterning forms the basis for algebraic reasoning, which is crucial for problem solving.

- The patterns that students explore at this stage are usually geometric, rhythmic or artistic.

Geometric Patterns

- Geometric activities involving sequences are created through the use of two- and three-dimensional shapes and other geometric ideas such as lines and points. Activity number 1, below, is an example of a geometric sequencing problem.

Activity Number 1

Use shapes to model the next two shapes in the pattern below.

- By extending the sequence and changing the orientation of the third triangle, the problem could be modified to present a greater level of challenge as outlined in activity number 2.

Which shape does not belong in the pattern below?

- Finally, students can be engaged in even more critical thinking by inviting them to consider the activity below.

Activity Number 3

Use cut-outs of shapes to model the next two shapes in the pattern below.

Rhythmic Patterns

- Rhythmic patterns are created using sounds and body movements. These can be done using the whole class with children being engaged either through individual or group participation.

- As an example, try this activity with your class: Use your hands to model the following actions for the class: 'snap – clap – clap'. As a whole group, have the class repeat the actions to create a pattern – make sure you change the pattern after it becomes apparent that students have become accustomed to it. Eventually, add another action to the pattern such as a 'stomp' and ask students to model various patterns such as, `snap – snap – clap – clap – stomp'.

- As students become familiar with these whole class activities, organise them into 3 groups and assign an action/sound to each group as follows:

  o **Group 1 – Snap**
  o **Group 2 – Clap**
  o **Group 3 – Stomp**

- Develop a rhythm that follows a pattern and ask students to carry out the actions in accordance with the rhythm. An example of a simple rhythm may be 'snap – snap – stomp – clap – clap'. Gradually make the pattern more difficult to follow until you have a pattern such as 'snap – snap – clap – clap – stomp – snap – clap'. Finally, place students in a ring and develop a pattern that they model by contributing the appropriate action/sound when it is their turn.

Other Types of Patterns

- Children also see many patterns in their environment that they can describe and explore:

    o The patterns created by tiles on the floor;

    o The design created by grills on windows and doors;

    o The patterns created on or with fabric such as a plaid or stripes.

Ordering or Seriation

- Ordering involves putting objects into a sequence based on some given characteristics such as size, length, width, colour, volume. Children may not be able to order objects at first, without some amount of trial and error; they may have to place the object randomly on the table and then move them from location to location until a suitable order is achieved.

- This skill is at a cognitively higher level than comparing and classifying as students have now moved from the creation of, for example, two distinct sets such as 'long' and 'short', to a rank arrangement built on the appreciation that even within the set of 'short' objects, some objects are taller than others.

- Ordering activities can be linked to children's literature. As you share the story of *Goldilocks and the Three Bears* with children, you may for example, have them order the bowls that Papa Bear, Mother Bear and Baby Bear own. Giving students cut-outs of bears and bowls is a good idea to get them to order the bowls and bears in some order of size.

- Additionally, ordering activities should be linked to knowledge of ordinal numbers – first, second, third, etc. Consider activity number 4 which follows.

Give groups of students about 3 similar size jars/bags filled to different degrees with counters. Play a game of 'Simon Says'. Give instructions such as:

- "Simon says take up the largest bag."
- "Simon says empty the largest bag into the smallest bag."
- "Simon says exchange 2 small bags with your neighbour."

Comparing, Classifying and Sorting

- Classifying and sorting involve the creation of sets based on similar features possessed by objects. This skill calls on students to explore the 'sameness' of objects and to create sets for those that are perceived to be the same in one respect or another.

- The skill of classifying is crucial for students to develop, as their ability to discriminate between members and non-members of a set is necessary for solving problems involving number types, number relationships and number principles.

- Comparing is related to classifying but focuses more on 'differences ' than on 'sameness'. In comparing, students look to see why elements cannot be classified together. Activities 5 and 6, below, show how classification tasks can be explored in the classroom.

Give students an assortment of objects – ensure that each object shares a characteristic with at least one other object (similar shape, colour, etc.). Give each group some bowls or transparent bags (ensure that bags are perforated to avoid suffocation) with a few objects already placed in them – one bag has 3 blue objects, another has 3 red objects, etc. Instruct students to finish packing away the objects on their desk in the plastic bags using the colour code already established.

Design a nesting tray: Glue about 3 empty transparent plastic bottles to a flat piece of board. Cut out a triangular, a circular or a rectangular hole in the cover of each bottle. Supply students with cut-outs of a number of different shapes – rectangles, triangles and circles. Make sure that each shape can fit through the cover with its corresponding cut-out. Instruct students to pack away the objects in the nesting tray.

Matching

- This is perhaps the most elementary of the skill set that students develop at the early numeracy stage; it is still quite crucial, however, if they are to reason about problems at later stages in the primary school curriculum.

- Matching, at the early numeracy level, describes students' ability to form one-to-one correspondence and thereby determine the number in a set. This forms the basis of the counting system and is also at the foundation for many problem solving tasks such as those that require sharing, increasing or decreasing a set.

Lower Primary

- At the lower primary level, students start learning mathematics in a **conceptual** manner – their observations of the world are associated with and explained by mathematical concepts and ideas. For example, they begin to associate large quantities with large numbers as they learn that numbers are made up of digits that have values determined by where they are placed in numbers.

- In addition, they use mathematical language such as 'certain', 'impossible' and 'maybe' to describe daily occurrences and are able to decide who received more sweets based on graphical representations of quantities.

- These ideas provide many opportunities for students to **move** from the **preparatory** to the **experiential** stage of problem solving. The experiential stage of problem solving is character-ised by:

  o A focus on problems that are drawn from the daily experiences that children are likely to have had. These problems allow children to imagine themselves in various circumstances or to apply real life ideas to generating solutions.

  o The use of problems that allow students the opportunity to reason about them using concrete representations or physical modelling.

  Consider the problem following which displays these two characteristics well.

Model the following for students:

- **Select a cut-out of a triangle similar to what is shown. Draw students' attention to the fact that it has 3 sides.**
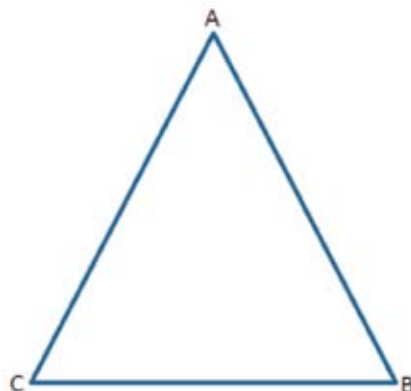
- **Fold over one edge (as shown by the dotted line below) to produce a figure with 4 corners.**
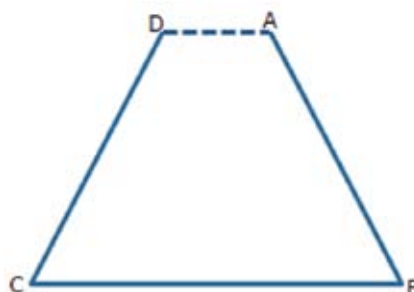
Provide students with a set of cut-outs (include cut-outs with as many as 8 sides). Guide them in carrying out the following instructions:

Instructions:
- a) **Select a cut-out of a shape with 4 corners. Fold over one corner and count the number of corners that it has now.**
- b) **Suppose a cut-out had 5 corners and one is folded over, how many corners would it have then?**
- c) **Figure out the same thing for cut-outs with**
    - i. **6 sides**
    - ii. **7 sides**
- d) **For any figure, how many corners does it end up with if one corner is folded over?**

- Some of the key skills that students should attain by the end of the lower primary level are outlined, by strand, as follows:

Number

Describing Number Relationships

- Many mathematical problems are based on numbers and number relationships. Understanding number relationships is thus a crucial skill in problem solving. Children at this stage are expected to be able to use their knowledge of numbers to solve problems involving number types, relationships and principles.

- Let us consider a problem which deals with number relationships:

Problem Number 2

An animal can have up to 8 legs:

- **Birds, etc. – 2 legs**
- **Dogs, etc. – 4 legs**
- **Insects, etc. – 6 legs**
- **Spiders, etc. – 8 legs**

As Noah stood at the door of the ark, he saw 16 legs go past him into the ark. How many creatures did he see? Find as many solutions as possible.

- Students' exploration of the problem may lead to any number of the following general principles about numbers being highlighted:

  - Some animals (birds, dogs and spiders), by themselves, can completely make up the 16 legs. This is as a result of the fact that division may sometimes result in a remainder and other times it may not.

  - The 16 legs could have been made up from different animals. In other words, different addends can be used to create the same sum.

  - Even if two children suggest that Noah saw the same animals going into the ark, the order in which Noah saw the animals may differ from one student to the next. In other words, the same addends can be used in a different sequence to give the same result.

  - Animals usually have an even number of legs as the total number of legs an animal has is usually twice the number it has on one side. In other words, if a number is doubled the result must be an even number

  - The more legs an animal has, the fewer of that animal could have been seen by Noah. In other words, large numbers when added together produce large sums quickly.

Operating on Numbers to Aid in Problem Solving

- As students become familiar with the four basic operations (addition, subtraction, multiplication and, to a lesser extent at the lower primary level, division), they are expected to move from simply performing algorithmic computation of mathematics tasks and move on to explore problems that are rich in their understanding of these operations.

- Compare the traditional (artificial) mathematics task in pane 1 to the mathematics problem in pane 2 below

PANE 1	PANE 2
27 + 6	Use the digits **7, 2, 6** to complete the addition problem below so that you get the smallest sum possible.

- The task in pane 2, while it results in the operation of addition being used, requires that students think critically about the following questions regarding addition and place value:

  - What determines the value of the sum obtained when two numbers are added?

  - Which place value position should have the largest/smallest digit?

  - How many different addition problems can be created from the 3 digits?

  - What general principles should be considered in creating the 2-digit number?

- In creating problems that involve operating on numbers, bear the following in mind:

  - Even though care is to be taken that students do not simply carry out operations on numbers, carefully selected and designed problems can aid students in developing their computational fluency. Such problems usually ask students to choose carefully how they create computational problems in order to achieve a specified outcome. Consider the following problems, for example:

Fill in the boxes with the digits 0, 6, 7, 7, 7, 9 to make a true equation.

$$\begin{array}{ccc} \square & \square & \square \\ - \quad \square & \square & \square \\ \hline & 1 & 0 \end{array}$$

Below is a 5 x 5 square. How many 2 × 2 squares can you find whose numbers give a sum of 10? One square (highlighted in yellow) has been found for you – note that

$S + T + X + Y = 3 + 2 + 1 + 4 = 10$.

A	B	C	D	E
2	6	1	4	3
F 5	**G** 3	**H** 0	**I** 2	**J** 1
K 1	**L** 1	**M** 4	**N** 5	**O** 2
P 2	**Q** 6	**R** 5	**S** 3	**T** 2
U 3	**V** 4	**W** 2	**X** 1	**Y** 4

In the table above, a pair is formed across (such as A + B) or down (such as A + F).

A	B	C	D	E
3	7	2	9	1
F 4	**G** 6	**H** 3	**I** 8	**J** 6
K 1	**L** 8	**M** 4	**N** 5	**O** 3
P 5	**Q** 1	**R** 2	**S** 7	**T** 6
U 5	**V** 9	**W** 7	**X** 4	**Y** 2

Horizontal pair

Vertical pair
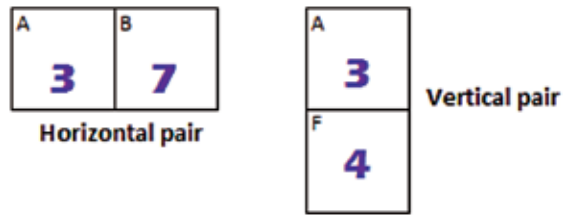
Slanted pairs (such as A + G) are <u>NOT</u> allowed:

1. **There are nine different pairs with a sum of 9. Can you find them and write them in the boxes below?**

Diagonal pair
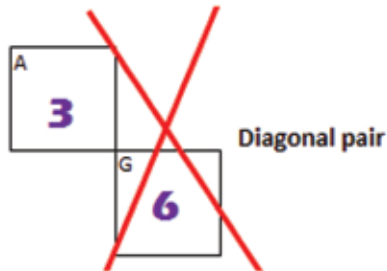
2. **All pairs have totals between 3 and 17, except 2. Can you determine which 2 they are?**

- Children at the lower primary level are not likely to have developed sufficient fluency in division and, to a lesser extent, multiplication to solve problems that require the application of these operations. Try as best as possible, therefore, to limit the operations-based problems to those that require thorough understanding and high degree of proficiency in addition and subtraction and just a conceptual understanding of multiplication and division.

- Most students will be able to add and subtract quite proficiently because they simple apply a mechanical procedure that always produces the correct answer once applied properly. However, they often do not understand various aspects of the algorithm and how it determines the answers that it produces.

  o This can be changed by creating what is called **parameters**. Parameters reduce the options that students have by telling them either how to construct a problem or the features of the solutions.

  o This converts traditional tasks into mathematics problems. Consider the following, for example:

Create an addition problem using the digits 2, 3, 4, 5, 9. What is its sum?

o   This problem asks students to do very little critical thinking. Suppose, however, the problem was posed as follows:

Problem Number 6

Use the digits 2, 3, 4, 5, 9 to create:

a) An addition problem for which the sum has an 8 in the tens place;

b) An addition problem for which the sum is less than 600 and has a 5 in the ones place.

Measurement

Iteration and Proportional Reasoning

- The measurement of a large object can be determined by using a small object repeatedly. A pencil, for example, can be placed end to end repeatedly to determine the length of a desk or table.

- When children are given pieces of string, or are told to use their hand-spans to determine the length of their desks, they are actually using the skill of iteration. Children need to have the skill of iteration in order to solve problems as very few objects that they explore will have a measure of exactly 1 unit.

- This idea of iteration must also be extended to an understanding of proportionality; students must understand that the smaller the unit being used to carry out the measurement, the greater the number of iterations needed to measure an attribute. A regular sized pencil, therefore, would require fewer iterations to cover the length of a table than the index finger would need.

- This idea becomes useful as students discover that efficient measuring seeks to minimise the number of iterations; therefore, some units are more appropriate than others in a specific measuring situation.

- In the problem, which follows, for example, children call upon their skills of iteration and pro-portional reasoning in order to help them reach a solution:

Give students the following items to use as units of measurement:

- **A decimetre strip (a 'long' from the base ten pieces)**
- **A strip of paper about 30 cm long**
- **A small paper clip**
- **A piece of string about 15 cm long**

Strip of paper

Decimetre rod

String

Paper clip

Place students in groups of 2 – 3 and tell each group to measure the length of their desk stating the answer as accurately as possible. Encourage them to use smaller pieces to measure any part that is less than a complete unit of a long piece.

Applying Principles of Conservation

- Conservation is the idea that various measurable attributes of an object (such as its height, mass, width, etc.) may not change if the object is changed in one or more of the following ways:

  - **Object is repositioned** : for example, a pencil is just as long even if it is turned upside down;

  - **Object is divided:** the space covered by a piece of cloth is the same if the cloth were torn into two pieces;

  - **Object is transformed:** the volume of water remains the same if it is stored in a tall narrow container or a flat wide container.

- The ability to apply principles of conservation to problems is one of the important outcomes of problem solving for children at this level of the curriculum, as this allows them to form mental pictures of 2- and 3-dimensional shapes from different orientations and perspectives.

- Consider the following problem which shows how the idea of conservation can be applied:
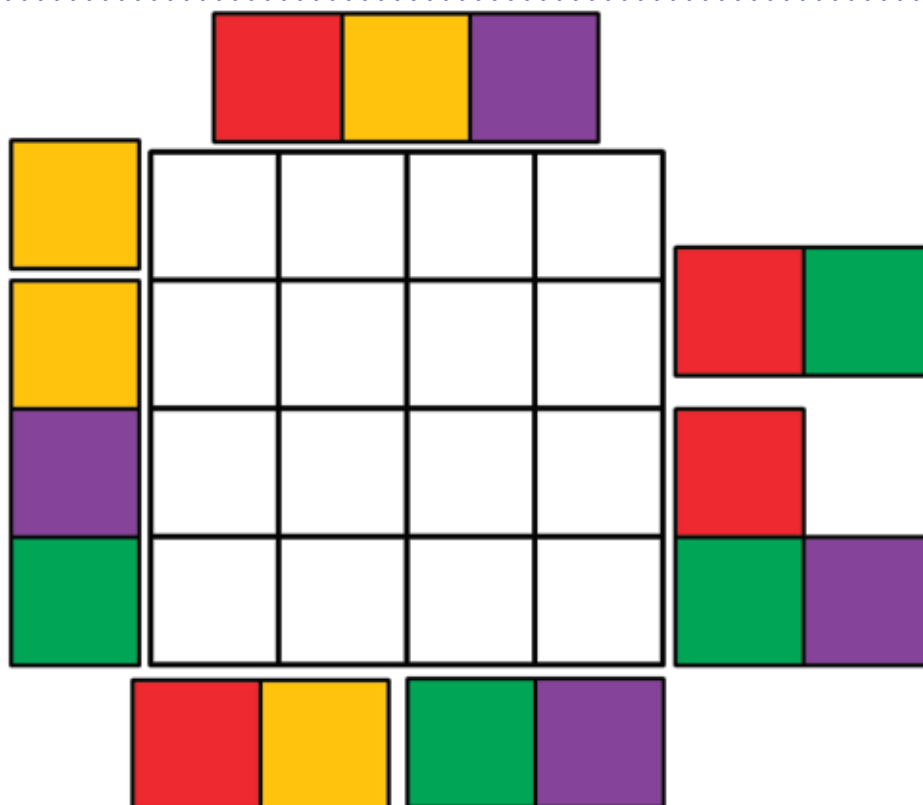
Give students the 4 × 4 grid above and cut-outs of the coloured pieces to the sides. Tell students that their challenge is to arrange the pieces to cover the entire grid but no row or column should have the same colour twice.

Estimating

• Students should be able to give approximate and appropriate estimates of quantities to be measured. This skill is developed in tandem with benchmarking, the process of establishing known reference points for common measures such as a metre, a kilogram, a litre, etc.

• Many of the Fermi problems (see *Problem Solving Handbook*) are useful for developing students' problem solving skills. Fermi problems are very open tasks in the sense that much of the information to solve the problem is not known and must be assumed or estimated by the students. These tasks provide students with an opportunity to systematically apply real ideas to mathematical tasks to provide, at best, a rough guess as an answer.

• A classic Fermi problem, for example, is given as follows:

About how many of your handprints would be needed to cover the floor of your living room?

- This problem allows children to break up the task into manageable parts in order to determine the overall answer. A child may, for example, determine the approximate number of tiles in their living room and, from there, determine how many handprints would cover a single tile and ultimately how many would cover the entire floor.
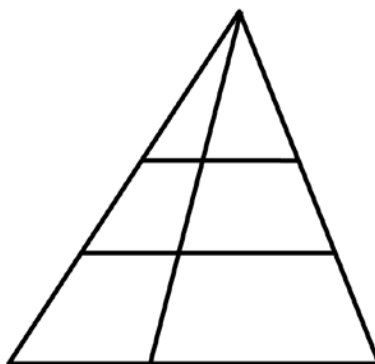
Geometry
Analysing Shapes and Diagrams Based on Geometric Ideas

- Students should be able to use the properties and features of shapes to analyse and explore them. This involves:

  - taking shapes apart; such as creating triangles from rectangles;
  - joining shapes to create new ones;
  - identifying symmetry in plane shapes;
  - stating basic relationships among the features (angles, lines, vertices, etc.) of a shape;
  - identifying shapes that are different from or similar to others within a set.

- An example of this skill being used in problem solving is shown in the problem below:

Problem Number 10

**Look at the shape below. How many triangles do you see?**

Spatial Reasoning

- Spatial reasoning describes the ability to manipulate and explore 2- and 3-dimensional shapes mentally. This skill allows students to visualise the result of changing (size, position, shape, orientation, etc.) of a geometric shape without or before actually doing so.

- Young students with spatial reasoning are also able to follow instructions given orally using such words as 'below', 'above', 'beside', 'middle', 'near', 'far'.

- Spatial reasoning takes into account the properties of 2- and 3-dimensional shapes but is developed primarily from experience working with shapes. At a very early age, students employ their spatial reasoning to solve problems similar to the ones shown below.

Problem Number 11
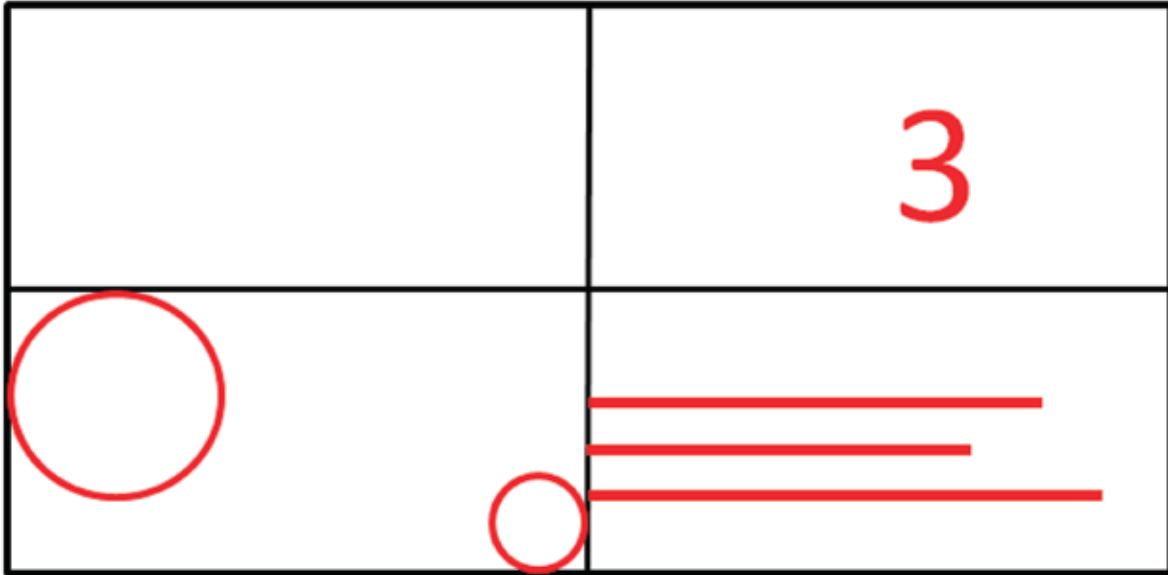
Last night you drew the picture above in your book. Now you have to describe to your friend over the phone what the drawing looks like so he/she can draw it as well. He/she cannot see your drawing so you have to be as precise as possible.

Problem Number 12
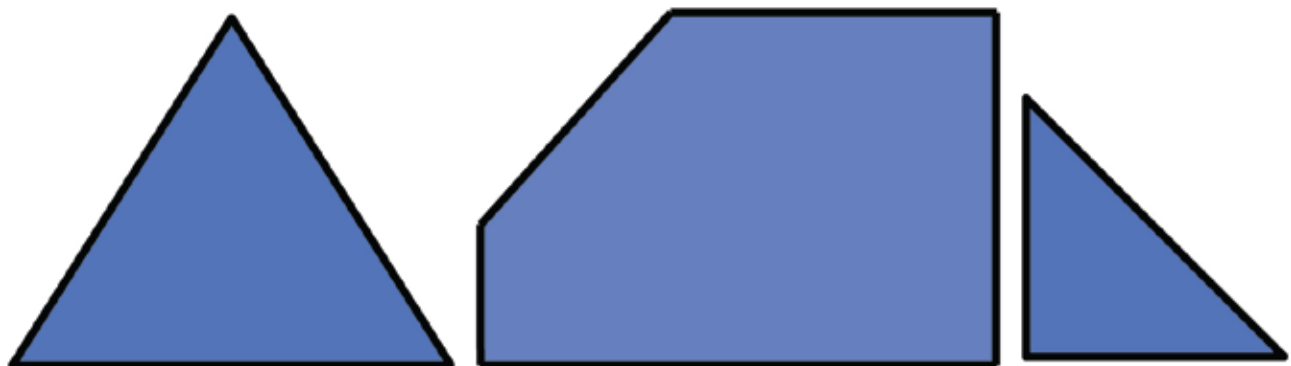
Cut out the above pieces and arrange them to get the following shape:

Statistics

Making Inferences and Predictions From Data

- As a problem solving skill, this involves:

    o   Collecting data from a variety of sources;
    o   Sorting data into distinct groups;
    o   Representing data using the appropriate tables, charts and graphs;
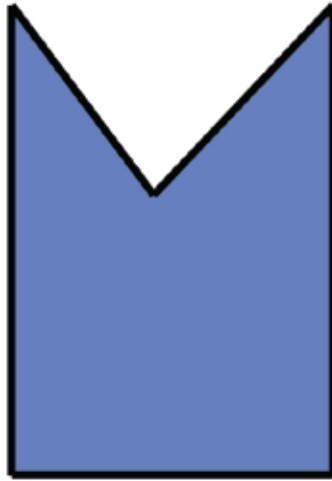   o   Interpreting tables, charts and graphs to obtain information;
    o   Use information obtained from data collected to determine if a situation is 'likely', 'certain' or 'impossible';

- Problems in statistics can be linked to other strands, particularly number, as is demonstrated by the problem below:

Problem Number 13

You have the following numbers in a bag: 1, 3, 4, 5, 7 and you draw 4 numbers, one after the other. As you draw a number, you write it on one of the lines below to create the largest four-digit number possible with them.

_____   _____   _____   _____

a) **If the first number you draw is 5, on which line would you put it and why?**

b) **If after the third draw you have 1 and 7 left in the bag, what is the best arrangement of the numbers you have drawn so far? Why?**

c) **If after 3 draws you have 5 and 7 left in the bag, what is the best arrangement of the numbers you have drawn so far? Why?**

Algebra

Patterning

- Patterns describe lists of objects, numbers, sounds, etc., that change in a precise way according to a given repeating rule. In algebra, students use algebraic symbols to describe and explore patterns and sequences.

- For example, consider the numeric pattern below:

$$3, 5, 7, 9, \underline{\hspace{1cm}}, \underline{\hspace{1cm}}$$

- Clearly, each new number in the pattern is generated by adding 2 to the preceding number, hence, the pattern is continued as follows:

$$3, 5, 7, 9, \mathbf{11}, \mathbf{13}$$

- The relationship in this pattern can also be described algebraically as follows, for any starting number, n:

$$n, n + 2, n + 4, n + 6, n + 8, n + 10$$

- Using this idea students can solve a problem as follows:

<div style="border: 1px dotted">

Problem Number 14

</div>

Today is October 3 and it is a Monday. You check the calendar and see that October 10 and 17 will also be a Monday. What day of the week will it be 35 days from today, October 3?

- In looking at the problem, students can be scaffolded into seeing that:

  o dates that have a difference of 7 (such as Oct. 3 and Oct. 10, Oct. 10 and Oct. 17) fall on the same day of the week;

  o for any day that falls on the date,

$$n, n + 7, n + 14, n + 21, n + 28, n + 35$$

  will all fall on the same day of the week.

Generalising

- Generalising describes the ability to look at specific cases to make general statements for all cases. When students generalise, they look at a few instances of an occurrence or a relationship and make conclusions about cases they have not seen before.

- A problem that involves the skill of generalising is shown below:

Draw a rectangle and draw a line through it, as follows:
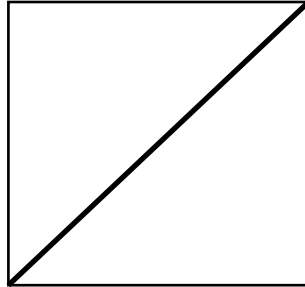
a) **How many regions are created?**

b) **Draw another line through the rectangle, making sure it does not cross the line that you have already drawn.**
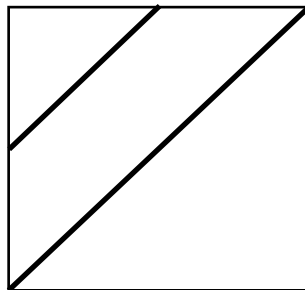
c) **How many regions does the rectangle now have?**

d) **Suppose you draw a third line through the rectangle, making sure the lines do not cross, how many regions are created?**
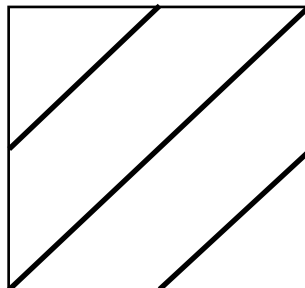
e) **Continue drawing lines through the rectangle and counting regions until you have reached 6 lines.**

e) Complete the following table:

Number of lines	Number of regions
1	2
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	

f) Explain how you are able to determine how many regions will be created for any number of lines drawn.

Upper Primary

- At the upper primary level, students move on from experiential problem solving to tackling problems that are more **abstract**. These problems are based on students' appreciation for contexts that are not readily modelled or relatable to real life experiences; as a result, to solve these problems, it is likely that students will have to draw on their mental processes and application of other concepts and not on their experiences.

- Many of these problems may be highly simulated or contrived; they bear little or no likeness to problems that students are likely to contend with, but they promote deep critical thinking and reasoning.

- A magic square problem, for example, (which follows), is unlikely to prove useful to children in their daily lives; however, it fosters the development of the computational skills that are to be promoted in the mathematics classroom.

Below is a 3 ×3 square.
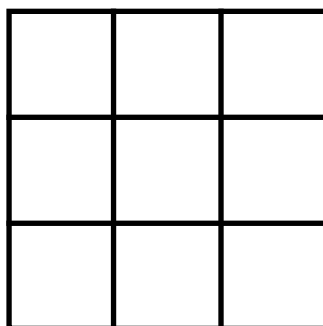
a) **Insert the numbers 1 – 9 in it so that the sum of each row, column and diagonal is the same.**

Such a completed square is called a magic square and the sum you obtain for each column, row and diagonal is called its magic number.

b) **Now try to create a magic square using the numbers from**

    i. **2 – 10**

    ii. **3 – 11**

c) **Develop and explain a system for completing any 3 × 3 magic square, even if the numbers are not in counting or any sequential order.**

- At the upper primary level, many of the problem solving skills that children are to develop are similar to those that should have been developed at the lower primary level. Given the differences in the developmental levels of students at the two stages, however, it is expected that students will engage in activities that promote different types of exposure to these skills at each level.

- The problem solving skills that students are to develop at the upper primary level (organised by strands) are outlined below.

Number

Describing Number Relationships

- Students at the upper primary level are expected to use the number relationships, properties and computation strategies to solve problems.

- Students should also be able to identify different types of numbers and the properties of these number types:

- o Odd and even numbers
- o Square numbers
- o Prime and composite numbers
- o Whole and decimal numbers

- Finally, multiplies and factors are important ways in which numbers are related; students should be able to identify the factors and multiples of a number and should have effective strategies for finding the multiples and factors of a number.

- The problems below show how number relationships, number types and their properties can be used effectively in the problem solving classroom.

Problem Number 17

Some numbers have all their factors, except 1, even. For example, the factors of 8 are 1, 2, 4, 8; except for 1, all factors are even.
  a) Make a list of the first 5 numbers that have all their factors, except 1, even.
  b) In general, which set of numbers will always have this property?

Problem Number 18

In a knockout quiz competition, losers drop out of the competition and winners advance to the next round until only one team remains. If there is an odd number of teams in any round, then the best loser from the previous round advances; this team is said to be given a bye.
  a) If there are 20 teams in the quiz competition,
      i.   In which round will the first bye be needed?
      ii. How many byes will be needed in total?
  b) List some numbers for which no byes will be needed.
  c) In general, for what set of numbers will no byes be needed?

Operating on Numbers to Aid in Problem Solving

- By the upper primary level, children have been exposed to the four basic operations and can now solve problems that require the use of any or all operations.

- The aim, once again, is to create problems for which students do not simply engage in repeated application of an algorithm, but rather reason about the process involved in their computation or the answers they obtain.

- The following problems demonstrate how students can be made to solve problems by using their computational skills.

Problem Number 19

The date is October 28, 2011. It is written as 28/10/11 and it is a special day because we can use the digits that make up the date to create a true mathematical sentence:

$$2 + 8 + 1 + 0 = 11$$
$$11 = 11$$

Alternatively, we could write the date as 28/10/2011 and in which case the mathematical sentence becomes:

$$2 + 8 - 1 = 20 - 11$$
$$10 - 1 = 9$$
$$9 = 9$$

a) Is your birthday special? Show how.

b) Can you think of other special days and show why they are special?

Problem Number 20

Michelle rolls a single die ; one side is face down and those dots cannot be seen. The sum of the dots that she can see, however, is 17. What number is faced down?

Suppose the sum of the number she sees when she rolls two dice is 30, what numbers are face down?

Proportional Reasoning

- Proportional reasoning refers to students' ability to compare two quantities to identify:

  o **Proportional changes:** changes in two quantities that result in the same relationship existing between them before and after the change. For example, if the number of children in a class doubles, then the number of desks to seat them must also be doubled.

  o **Covariation:** changes in quantities that coincide with each other. Does an increase in one quantity cause a decrease or an increase in the other quantity?

  o **Ratios:** proportional reasoning is represented by ratios that show the relationship that quantities have with each other in reference to a total.

  o **Part-whole relationships:** a quantity can be expressed as a part or portion of the total and written as a fraction.

- Some of the topics that are important for the development of proportional reasoning are:

  - **Fractions**
  - **Percentages**
  - **Multiplication and Division**
  - **Ratios**
- Following are two problems that outline the importance of proportional reasoning.

Problem Number 21

Marlon is having a party tomorrow. He has already made all the plans; he has already invited the guests and ordered the amount of orange juice he thinks will be needed.

He is having second thoughts now and wants to change the number of persons who attend OR the amount of orange juice he provides OR both. Complete the table below by indicating whether EACH person will get 'more', 'less' or the 'same' amount of orange juice in each case. Is there any case for which you are not sure? Why?

		Amount of Orange Juice		
		Increase	Decrease	No change
No. of Guests	Increase			
	Decrease			
	No Change			

The large outer square, PQRS, below represents 1 whole unit. It has been partitioned into pieces. Each piece is identified by a letter.
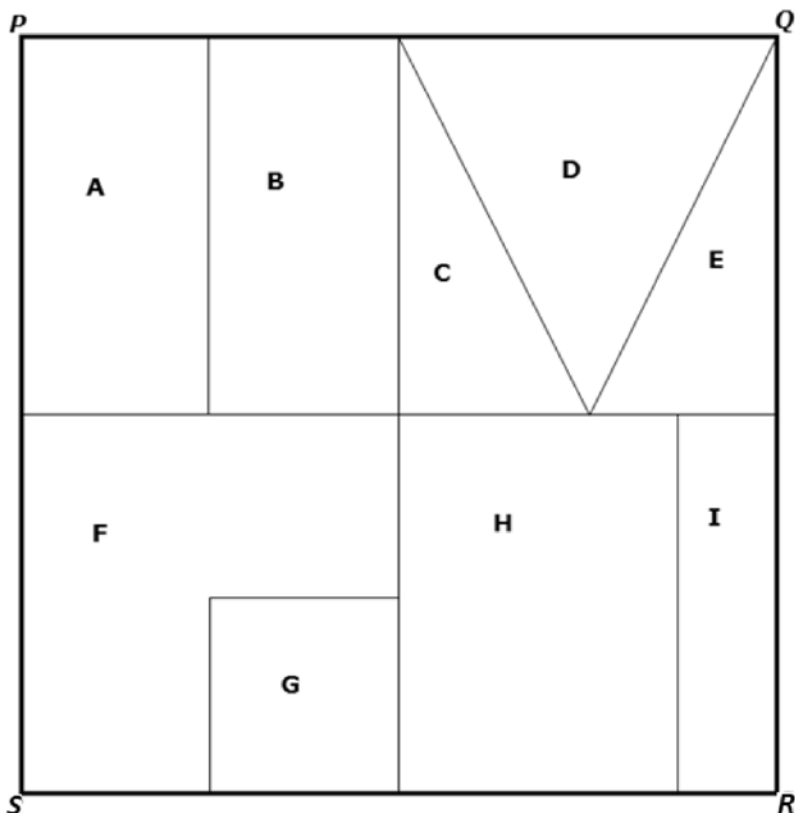
a) Decide what fraction each piece is in relation to the whole square and write that fraction on the shape.

b) Explain how you know the fractional name for each of the following pieces.

    i.   A

   ii.   C

  iii.   D

  iv.   F

c) Identify a piece or a collection of pieces from the square that will give you an amount close to

    i. $\dfrac{1}{5}$

   ii. $\dfrac{2}{3}$

d) Design your own fraction square. Make sure you include a

    i. $\dfrac{1}{5}$

    ii. $\dfrac{1}{3}$

Measurement

Applying Principles of Partitioning and Transitivity

- Partitioning describes the division of a unit of measurement into smaller parts. In the metric system of measurement:

  - when small units get large, they are renamed and capsized into a single large unit. This means that large units are made up of other smaller units; the result is that one unit can be expressed as a fraction of or a multiple of another. For example, a centimetre (cm) is one-tenth of a decimetre (dm) and is also 10 times a millimetre.

  - A measure can be broken up into parts and stated using different units. A height of a man 1.78 meters tall, for example, can be stated as:

  1 m + 7 dm + 8 cm

  - Given the principles of renaming units, relationships among units are determined by powers of ten – one unit is either 10 times, or 100 times or 1,000 times more than another unit. This allows us to apply place value ideas to measurement; this is illustrated in the diagram below, which shows the relationship between cm, dm and m.
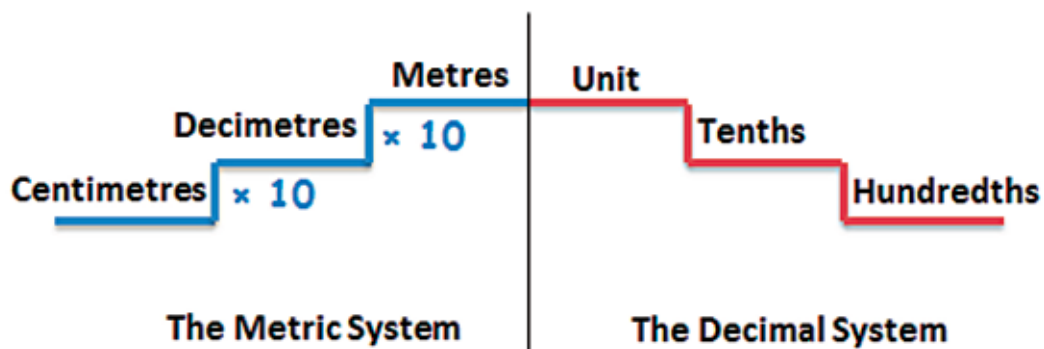
- Partitioning allows us to measure with greater precision, as if an attribute (such as mass, length or volume) is not a full unit; then its measure can be stated using fractional units. A man with height less than 2 meters but more than 1 meter, may be stated as 1.78 metres or 1 metre and 78 centimetres.

- Finally, the transitive property of measurement allows us to compare two objects by considering their relationship to a third object. For example, if John is taller than Mary and Mary is shorter than Paul, then John must also be taller than Paul.

- Also, since 10 cm =1 dm and 10 cm = 100 mm, then 100 mm = 1 dm; the transitive property allows us to use one object as a referent point when comparing two quantities.

- A problem that students can solve by applying ideas related to partitioning and transitivity is shown below.

Problem Number 23

Give students the following strips of paper.

- **A – 32 cm**
- **B – 16 cm**
- **C – 8 cm**
- **D – 4 cm**
- **E – 2 cm**

Instruct them to answer the following questions:

a) **State the length of A in terms of B and C.**

b) **State the length of A using at least 3 other pieces.**

c) **Use the pieces to measure the length of your desk giving the answer as accurately as possible.**

d) **State the length of your desk using at least 3 different pieces.**

Applying Measurement Ideas to Real Contexts

- Students should appreciate that measurement is carried out on attributes and phenomena that are real and observable:

  o Shapes and spaces have length and width and, as a result, area and perimeter as well;

- o   Solid objects have mass, depth and volume;
- o   Containers have capacity;
- o   Time is divided into measureable portions such as seconds, minutes, hours, days, etc.

- For students to appreciate the fact that attributes are related to objects that can be touched, manipulated or transformed, the link between geometry and measurement must be emphasised.

- By emphasising the relationship between geometry and measurement, students observe that though they may be measuring the same attribute, different (but related) procedures may be needed if context changes.

- For example, area as an attribute of space is measured differently depending on the shape of the space being considered. However, since area is essentially the counting of squared units contained within a space, the different procedures are closely related.

- This idea is highlighted through the use of rectangles and triangles in the example which follows:
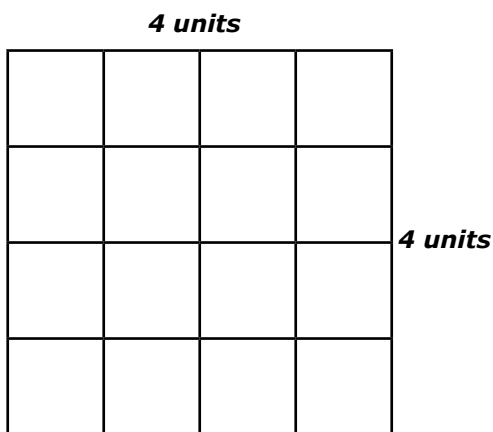
4 units

4 units

- **A rectangle with length (l) of 4 units and width (w) of 4 units;**

- **There are 16 small squares contained in the rectangle and therefore area of rectangle = 16 square units;**
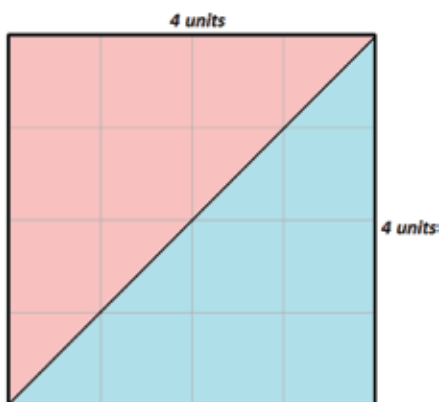
4 units

4 units

- **A similar rectangle to one drawn above – l = 4 units and w = 4 units**

- **Rectangle has been divided into 2 triangles**

- **Area of each triangle = 8 square units**

- **Hence, area of triangle is half the area of rectangle**

- Students should also be able to appreciate that changing an object's dimension affects attributes differently; some attributes may not change (they are invariant) if the object changes while others will. Area and perimeter provide many useful examples of the effect changes have on attributes. Consider the shape below, for example, which has area of 12 square units and perimeter of 14 units.
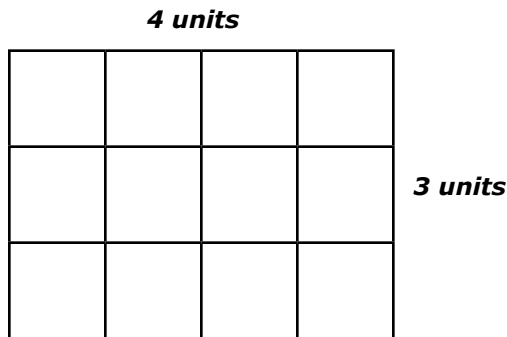
4 units

3 units

- **Perimeter = 4 + 3 + 4 + 3 = 14 units**
- **Area = 4 × 3 = 12 square units**

- By slightly changing the appearance of the rectangle above, the shape below is produced. It too has an area of 12 square units, however, its perimeter is 16 units.
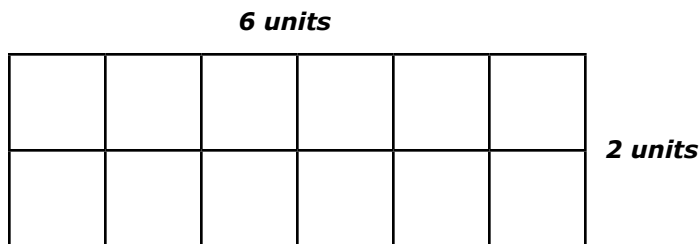
6 units

2 units

- **Perimeter = 6 + 2 + 6 + 2 = 1 units**
- **Area = 6 × 2 = 12 square units**

- By exploring further, students come to appreciate that:

  o Objects with different dimensions and perimeters may have equal areas .
  o Objects may have equal perimeters but different areas.

- The following problem shows how an appreciation for the link between ideas in geometry and measurement can be used in problem solving.

....
Problem Number 24
....

Pauline has 16 meters of wire fencing to make a rectangular garden.

    a) **What are the possible dimensions of her garden?**

    b) **What is the largest area that Pauline's garden may have?**

    c) **What is the smallest area that Pauline's garden may have?**

Geometry

Spatial Reasoning

- Students use their spatial reasoning skills when they:

  - Visualise the result of growing or shrinking an object proportionally.

  - Form mental pictures of the result of changing the orientation of an object .

  - Imagine what will be created by combining or partitioning objects.

  - Discriminate between objects based on size, dimension or orientation.

- The development of spatial reasoning is supported by an understanding of ideas such as:

  - **Symmetry:** some shapes can be divided into halves that are mirror images of each other.

  - **Congruence:** objects may have the same shape and size but are in different positions or orientation.

  - **Similarity and proportionality:** if each feature of an object is enlarged or reduced in the same way then the new object formed is proportional and similar to the original object.

- Students use spatial reasoning to solve problems such as the one which follows:

Give students 2 sets of cut-outs of the following shapes.
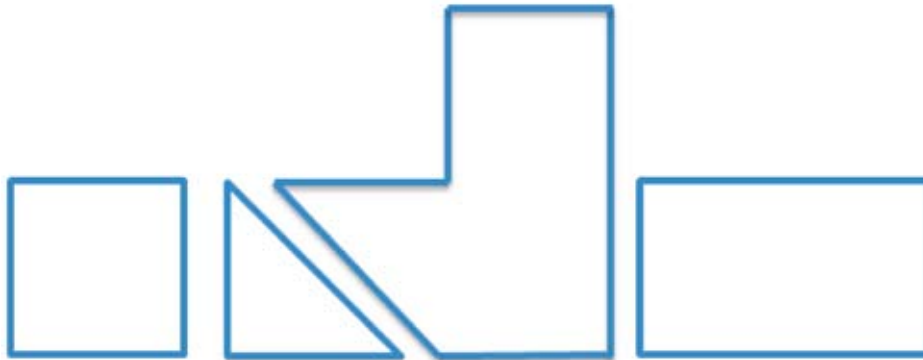
Give students a print out of the following shapes.
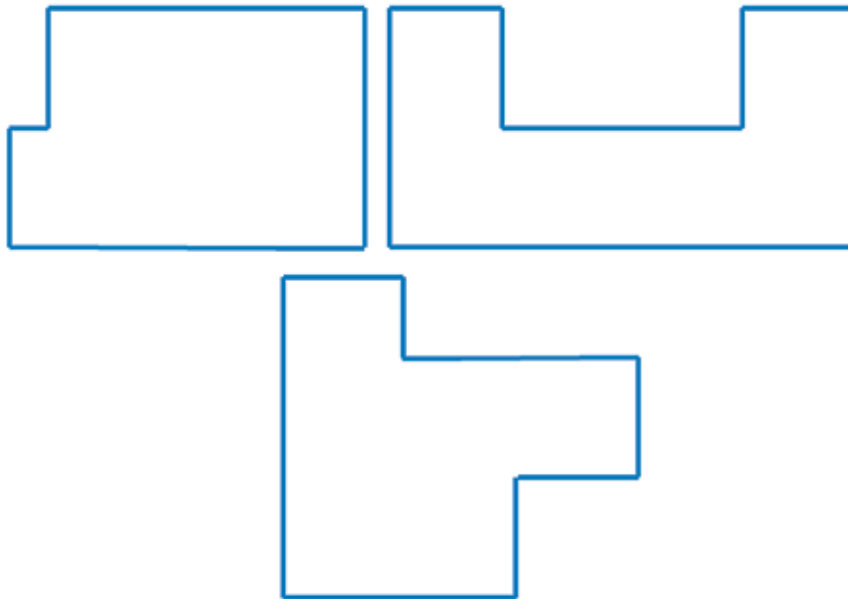
Tell students that various pieces of the cut-outs can be combined to create the shapes on the print out. Challenge them to create each shape on the print-out using the shapes on the cut-outs.

- A major application of spatial reasoning that students should demonstrate is tessellation. Tessellating is the repeated use of a shape to cover a surface such that there are no overlaps or gaps. Tessellating makes use of the following ideas:

  - **Area**
  - **Congruence**
  - **Angles**
  - **Symmetry**

- A typical tessellation problem that students may be asked to solve is shown below:
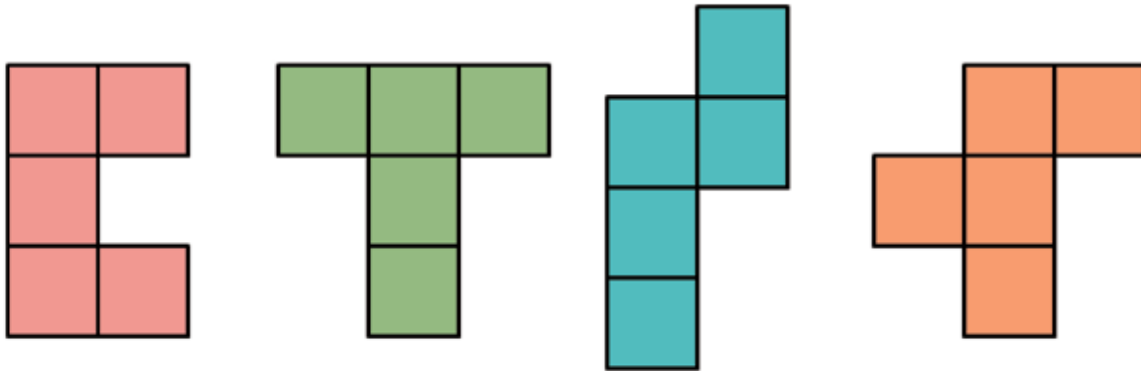
A pentomino is a shape made by joining 5 squares of the same dimension. A pentomino is like a domino, except that while a domino has 2 squares, a pentomino has 5. Below are 4 examples of pentominoes.

Can you use the pentominoes to tessellate the region below? Use each piece as often as you want.

Identifying and Describing Geometric Relationships

- As students work with shapes, they should observe that special relationships exist among them. They should also observe that shapes have certain characteristics that relate to each other in a predictable manner.

- Students should be able to describe these relationships and use them to develop arguments and solve problems in geometry.

- Consider the problem, below, for example:

Problem Number 27

- **Regular polygons have sides that are equal in length; they also have angles of equal size.**
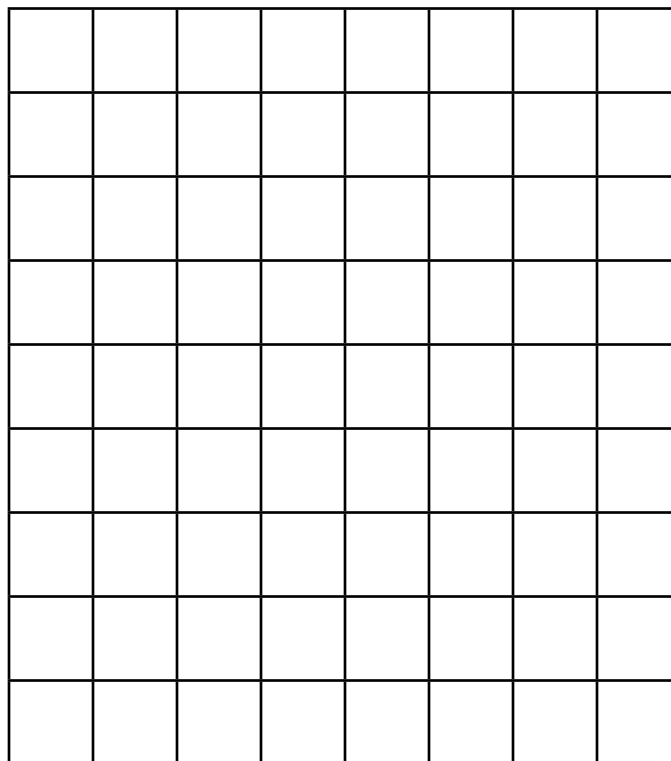
- **Equilateral triangles and squares are examples of regular polygons. How many sides does a regular polygon have if it has exactly 20 lines of symmetry?**

- **By exploring regular polygons with only a few sides, students come to appreciate a key relationship. This is outlined below:**

Equilateral Triangle	**Square**	**Regular Pentagon**
3 sides	*4 sides*	*5 sides*
3 lines of symmetry	*4 lines of symmetry*	*5 lines of symmetry*
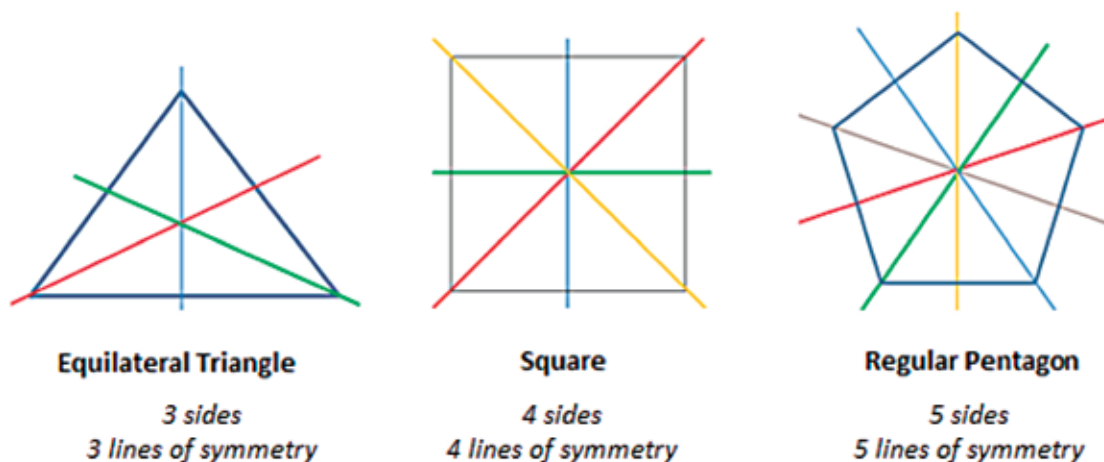
- From this exploration, students come to conclude that the number of lines of symmetry that a regular pentagon has will be equal to the number of sides that it has. From here, students can argue that a polygon with 20 lines of symmetry would have 20 sides.

Statistics

Selecting Appropriate Statistical Tools

- As students work with data, they are exposed to many ways of displaying and analysing the information they produce. They create graphs and charts and must make sense of data using measures of central tendency such as mean, mode and median.

- While it is crucial that students understand how to apply the various statistical techniques that they learn and how to construct the various graphs and charts that they are exposed to, it is perhaps even more important that they know when it is appropriate to apply each one.

- The problem below shows how an understanding of statistical tools such as mean, mode and median can be used in problem solving.

Problem Number 28

Hugh, Rita and Sandy were trying to see who could make the closest estimate of when a 30-second interval had passed. They each took 5 turns guessing while someone else held a stopwatch. The time shown on the stopwatch for each attempt by each person is shown below:

Hugh

31 seconds   25 seconds   32 seconds   27 seconds   28 seconds

Rita

37 seconds   19 seconds   40 seconds   36 seconds   22 seconds

Sandy

32 seconds   38 seconds   24 seconds   32 seconds   32 seconds

a) **Who do you think is best at estimating 30 seconds?**
b) **Give reasons for your answer.**

Applying Ideas of Probability to Daily Occurrences

- Students understanding of probability should allow them to apply the ideas of 'certain', 'impossible', 'likely' and 'unlikely' to game scenarios to say whether they are fair or unfair. They are also expected to carry out experiments and determine the probability of all outcomes.

- In the problem which follows, students are asked to decide on the fairness of a game based on how likely it is that either player can win.

Problem Number 29

Keisha and Shawna are playing a game. They roll a pair of dice and find the sum of the numbers rolled. If the sum of the numbers is 7, then Keisha gets 7 points; however, if the sum is NOT 7, then Shawna gets 1 point. Is the game fair? Why or why not?

## Algebra

Observing and Applying Functional Relationships

- This is perhaps the most important problem solving skill that students will develop during primary school. Since mathematic is the study of patterns, then the ability to observe, describe and apply these patterns is paramount in the mathematics classroom.

- Functional relationships exist when changes in one variable or quantity are determined by and can be predicted by changes in another variable or quantity. In this case, one variable is said to be a function of another.

- When a functional relationship is observed, then a general rule governing the relationship usually exists and this can be explored, deduced and stated using algebraic principles.

- Students use functional relationships to generalise, which is the use of specific cases to make conclusions about all cases, including those that have not been seen or explored as yet.

- When students generalise they talk about the nth case or the yth result, using algebra to ensure that their conclusions can be applied to any case.

- Students apply their knowledge of functional relationships and the skill of generalising when they

  o Determine from investigation that the area of a circle is πr^2;
  o Conclude that for any triangle, its three internal angles must sum to 180°;
  o Develop formulae for finding the area of various polygons;
  o Use algebraic reasoning to solve simple cases to determine the result for complex cases.

- General statements can be written in prose form or algebraically using symbols, even though meaningful scaffolding exercises should be provided for students to become proficient at generalising using algebraic symbolisation.

- In describing a relationship between money spent on snacks and money spent on juice, a child may conclude that:

  "I realise that the amount of money spent on snacks is always ten dollars less than the amount spent on juice."

- This conclusion could also be written using algebraic symbols as follows:

  "For $n spent on snack, $n – 10 is spent on juice."

- Below is a problem that requires the application of the skill of describing and applying functional relationships:

1. **Use matchsticks/toothpicks to create the triangle below:**
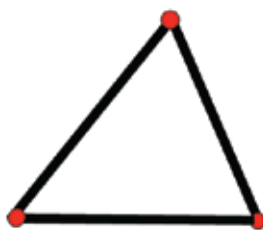
2. **Add two more matchsticks to add another triangle to shape you now have:**
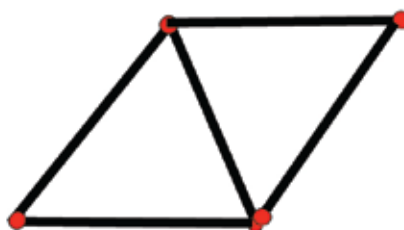
3. **How many matchsticks would be in a shape with**

    a) **3 triangles?**

    b) **4 triangles?**

    c) **5 triangles?**

    d) **89 triangles?**

    e) **n triangles?**

4. **A shape has 203 matchsticks, how many triangles does it have?**

•   By exploring simple cases, students come to appreciate that if they want to determine the number of matchsticks in a shape, then they should:

    "Double the number of triangles that the shape has and then add 1 to the answer."

•   This can be verified from the table below:

Number of triangles	1	2	3	4	5	8	10
Number of matchsticks	3	5	7	9	11	17	21

•   For a shape with n triangles, therefore, there will be 2n + 1 matchsticks. This can be used to predict the number of matchsticks in shapes with many triangles such as 89.

Chapter 8:
Standards

The Jamaican standards for mathematics are statements about what students should know and be able to do in order to meet the attainment targets of *The Revised Primary Curriculum (RPC)*. The standards are articulated by grade level and describe a connected body of mathematical understandings and competencies that provide a foundation for all students in Grades 1 to 6. The standards embody the process strands articulated in the *Jamaica Early Childhood Curriculum Guide* and articulate with those in the National Curriculum for Secondary Schools.

The RPC and the standards for mathematics complement each other. The standards provide support for teachers to monitor students' progress and the success of teaching and learning programmes. They provide administrators and other stakeholders with a comprehensive overview of what students should be achieving in all grade levels. Most importantly, they provide a means by which students' performance can be assessed in relation to the curriculum's attainment targets and objectives. Standards provide teachers with requisite information they will need to design instruction capable of meeting the needs of the students. Together, the curriculum and the standards will play an important role in the development of students' mathematical ability, thereby increasing their levels of numeracy.

The RPC promotes five mathematical content strands that are supplied by five process strands. The content strands reflect 'what students should learn', while the process strands reflect 'ways of acquiring and using content knowledge' to the extent that students can become mathematical problem solvers, communicate mathematically, reason arithmetically, make mathematical connections and use mathematical representations to model and interpret practical situations.

Why Mathematics Standards?

Mathematics standards to support the Revised Primary Curriculum are necessary as they play a significant role in the process designed to:

- improve classroom practice;
- guide teachers in ensuring that the appropriate content is covered;
- facilitate the development and allocation of resources (human and physical) to support students' learning;
- guide classroom and national assessments.

Mathematics Skills for Early Childhood Development

With the Ministry of Education's focus on **"*Every child can learn; every child must learn,*"** educators must seek to engage young minds in a variety of interesting ways to learn mathematics.

It is important therefore to teach mathematics to the young child because:

- math is a life skill;
- we need to lay the foundation for future learning of mathematics;
- children need to develop into logical thinkers;
- this is a prime time to get them interested in counting, sorting, building shapes, measuring and estimating;
- they need the opportunity to experience mathematics as they play and explore their world;
- linking mathematics with other disciplines can better help them learn the subject.

The process strands articulated in the *Jamaica Early Childhood Curriculum Guide* are similar to those in the *Revised Curriculum Guide* for grades 1-6. This is predicated on the notion that children learn best by doing.

In relation to the content identified for learning in the *Jamaica Early Childhood Curriculum Guide* there is much evidence that children are being prepared for more formal learning at the primary level. Skills developed at this stage such as comparing, counting, describing, playing with solids, and measuring are crucial in the quest to prepare students to appreciate and quantify the mathematical world around them.

Articulating Mathematics at Grade Six
With That at the Secondary Level

The mathematics standards and the *Revised Primary Curriculum* should assist in providing the secondary school system with students who have a strong mathematics foundation evidenced by knowledge of the strands and processes that are required for them to fully access the secondary mathematics curriculum. This will eventually redound, in the long term , to improved performance levels in the CSEC mathematics examinations after five years of secondary education.

The Structure of the Mathematics Standards

- Mathematics programmes in schools are based on the RPC.
- The National Assessment Programme and assessment instruments used in schools are aligned with the structure of RPC.

The standards are not intended to encompass the entire RPC curriculum for a given grade, nor prescribe how the content should be taught. Teachers are encouraged to go beyond the standards and to select instructional strategies (see Chapters 4-5) and assessment methods appropriate for their students.

International Benchmarking

The development of the mathematics standards and benchmarks was informed by a comparative analysis of standards for mathematics developed in Singapore, Hong Kong, Australia, the United Kingdom, Canada and the United States. In particular, several benchmarks from The National Council of Teachers of Mathematics (NCTM) were adopted.

It has become increasingly important to gauge Jamaica's expectations against international standards. By comparing our standards with international trends, it provides us with the opportunity to share in the best practices and learn from the successes of others. It affords us the opportunity to examine high performing models, such as the one used in Singapore, to gain new insights and explore possibilities for collaboration.

The mathematics standards are universal across the grades. Benchmarks are aligned to the standards. These are statements of expectations of the content and processes that are specific to the different grade levels. Following is a table showing content and process standards from Early Childhood to Grade Six.

The Content and Process Standards from Early Childhood to Grade Six

Content Standards	Process Standards
Number Students will: demonstrate an understanding of numbers, the number system and the relationships among numbers, as well as apply number concepts to compute fluently and solve problems.	**Problem Solving** Students will: develop new mathematical knowledge through problem solving and employ a variety of problem solving strategies to solve mathematical problems.
Measurement Students will: demonstrate an understanding of measurable attributes and the units, systems, and processes of measurement, as well as use direct and indirect measurement and estimation skills.	**Reasoning and Proof** Students will: use reasoning and proof when solving mathematical problems including (i)　　　making and investigating mathematical assumptions; (ii)　　　developing, selecting, evaluating and applying mathematical arguments.
Geometry Students will: demonstrate an understanding of (i) the properties of two- and three-dimensional geometric shapes; (ii) visualisation and spatial reasoning.	**Communication** Students will: express mathematical concepts and processes using clear and precise language, and be able to analyse and evaluate mathematical strategies communicated by others.
Algebra Students will: demonstrate an understanding of patterns and relationships, and represent and analyse mathematical situations using algebraic symbols.	**Connections** Students will: demonstrate how mathematical ideas are interconnected, and use the interconnections to apply mathematical processes to various contexts.
Statistics and Probability Students will: (i) solve problems involving the collection, organisation, display and analysis of data in practical situations; (ii) understand and apply basic concepts of probability.	**Representation** Students will: (i) create or select symbols, diagrams, tables, charts, etc. to organize, record, and communicate mathematical ideas; (ii) use mathematical representations to solve problems, and to model and interpret real life situations.

Chapter 9:

Taxonomy of Numeracy Competencies and Skills

Primary Level

In keeping with the 2015 vision of 85% numeracy attainment at the primary level, the Taxonomy of Numeracy has been developed outlining minimum competencies and skills that students at the different stages at early childhood up to the end of primary should demonstrate. Embedded in those statements of expectations are characteristics of the numerate behaviour such as:

(1) **Conceptual understanding** refers to the "integrated and functional grasp of mathematical ideas", which "enables them [students] to learn new ideas by connecting those ideas to what they already know." A few of the benefits of building conceptual understanding are that it supports retention, and prevents common errors.

(2) **Procedural fluency** is defined as the skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.

(3) **Strategic competence** is the ability to formulate, represent, and solve mathematical problems.

(4) **Adaptive reasoning** is the capacity for logical thought, reflection, explanation, and justification.

(5) **Productive disposition** is the inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Stages of Numeracy

The fluency and facility with which students master the various mathematics concepts and skills will differ from person to person. Consequently, the stages of numeracy are best presented as a continuum of attributes manifested throughout the entire development process of an individual.

It is within this context the age-grade boundaries of developmentally appropriate characteristics that these stages of numeracy development related to the primary level are identified:

- **Preparatory Stage – Early Childhood Level (0 – 5 years old)**

- **Exploratory Stage – Grade 1 (6 – 7 years old)**

- **Developmental Stage – Grades 2 & 3 (8 – 9 years old)**

- **Intermediate Stage – Grade 4 (10 years old)**

- **Perceptual Stage – Grades 5 &6 (11- 12 years old)**

The basic competencies and skills highlighted in each age-appropriate developmental stage are referenced against the five mathematical strands – Number, Measurement, Geometry, Algebra and Statistics and Probability in the RPC.

Descriptors of the Stages

Stage 1: Preparatory (Early Childhood)

At this stage children begin to explore the principles of counting, sorting and matching objects. The following are the critical developmentally appropriate competencies and skills associated with the Preparatory Stage.

Numeracy
Preparatory Stage
Early Childhood Level
Basic Competencies and Skills

- Repeat number names
- Write numerals and number names to at least ten
- Count in sequence from 1 to at least 10
- Use terms such as bigger, larger, smaller, longer etc.
- Distinguish between letters and numbers
- Replicate patterns and shapes
- Sort objects (by shape, colour, size)
- Add at least 2 groups of objects
- Know simple concepts such as time (day, event)

0 to 5 Years old

Stage 2: Exploratory (Grade 1)

At this stage the children are working with concrete materials, discussing mathematical ideas, using measurement terms, developing an awareness of shapes in the environment, making connections with mathematics and the real world. The following are the critical developmentally appropriate competencies and skills associated with the Exploratory Stage.

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Numeracy
Exploratory Stage
Grade 1

Basic Competencies and Skills

6 - 7 Years old

- Count sequentially up to at least 20
- Read and write numerals up to at least 20
- Know place value of numbers up to 20
- Add 2-digit numbers using 'tens' and 'ones' without renaming
- Divide sets of at least 20 into equal parts
- Identify shapes (circles, rectangles, squares) showing fractions, e.g. halves and fourths
- Tell time on the hour and half hour
- Identify dates on a calendar
- Estimate and compare measurements using terms such as shorter, larger or equal
- Identify basic geometric shapes (circles, squares, triangles, rectangles)
- Use addition and subtraction facts to solve for unknown e.g (n + 2 = 5) n = ___
- Read simple picotgraphs

Stage 3: Developmental (Grades 2 & 3)

At this stage the children are demonstrating an understanding of several mathematical concepts such as place value, face value, digit value, expanded notation. They can use measurement terms, name polygons using their end points, differentiate shapes in the environment, solve problems related to 'n' sentences, make predictions and interact with data using different types of graphs. The following are the critical developmentally appropriate competencies and skills associated with the Developmental Stage.

Numeracy
Developmental Stage
Grades 2 and 3
Basic Competencies and Skills

8 - 9 Years old

- Know the value of 2-digit and 3-digit numbers
- Expand 2-digit and 3-digit numbers
- Add and subtract whole numbers up to 3 digits
- Name numbers in different ways
- Know multiplication facts up to multiples of four
- Know money and the worth of coins
- Identify fractions ½, ¼, ¾, etc.
- Solve problems involving equivalent, proper, improper and mixed fractions
- Know simple measurements, their use and their symbols (e.g. L, m, cm)
- Read and measure temperature, time, distance
- Estimate the area using grids
- Measure distances around basic shapes
- Identify points, lines and right angles
- Name basic shapes
- Record data using tally and/pictographs

Stage 4: Intermediate (Grade 4)

At this stage students consolidate previously learned concepts and skills and are applying basic operations to solve more complex problems. The following are the critical developmentally appropriate competencies and skills associated with the Intermediate Stage.
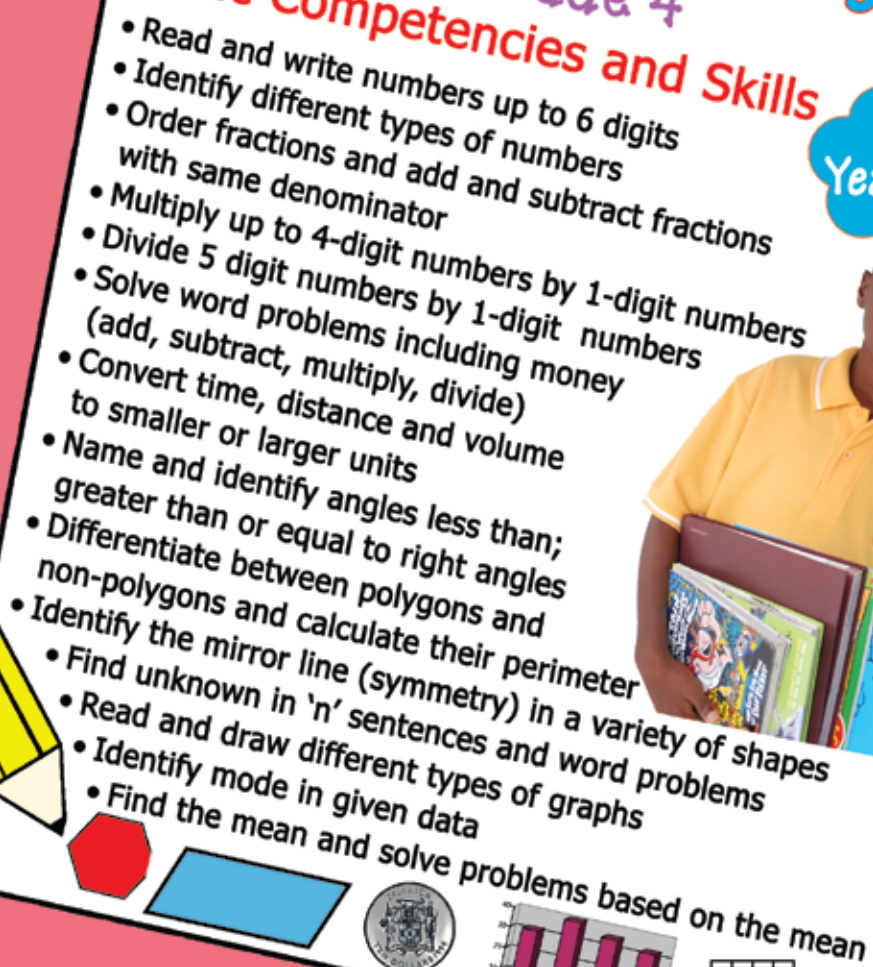
Numeracy
Intermediate Stage
Grade 4

Basic Competencies and Skills

10 Years old

- Read and write numbers up to 6 digits
- Identify different types of numbers
- Order fractions and add and subtract fractions with same denominator
- Multiply up to 4-digit numbers by 1-digit numbers
- Divide 5 digit numbers by 1-digit numbers
- Solve word problems including money (add, subtract, multiply, divide)
- Convert time, distance and volume to smaller or larger units
- Name and identify angles less than; greater than or equal to right angles
- Differentiate between polygons and non-polygons and calculate their perimeter
- Identify the mirror line (symmetry) in a variety of shapes
- Find unknown in 'n' sentences and word problems
- Read and draw different types of graphs
- Identify mode in given data
- Find the mean and solve problems based on the mean

Stage 5: Perceptual (Grades 5 & 6)

At this stage students continue to consolidate previously learned concepts and skills and are applying basic operations to solve more complex problems than at the Intermediate stage. The following are the critical developmentally appropriate competencies and skills associated with the Perceptual Stage.

Numeracy
Perceptual Stage
Grades 5 and 6

Basic Competencies and Skills

11 - 12 Years old

- Read and write numbers up to 8 digits
- Round numbers up to 8 digits
- Do computations with fractions, decimals, whole and mixed numbers and money
- Name and list members of a set
- Identify multiples and factors of given numbers
- Write numbers in expanded form
- Write equivalent ratios
- Solve problems involving percentages of numbers and amounts of money
- Compute area and perimeter of regular and irregular polygons
- Interpret and use scale drawings
- Know the relationship between the different parts of a circle
- Tell time using 12-hour clock
- Identify, describe and state properties of regular and irregular shapes
- Identify different types of triangles and angles
- Identify different types of lines (e.g. parallel, perpendicular)
- Identify and classify solids based on faces, edges and vertices
- Solve mathematics sentences using algebraic expressions
- Use algebraic expression to describe word problems
- Solve simple equations
- Identify patterns in data
- Calculate mean and median
- Determine probabilities
- Make inferences and interpret data using different types of graphs

Chapter 10:

School Accountability

The Ministry of Education aims at transforming education from a system where the measurement of educational outcomes against inputs is strategized to improve school performance and ultimately students' numeracy achievement. In order to achieve this desired result, in 2008, a system of accountability was implemented. Successful operation of the system of accountability will enhance sound decision making and effective use of resources. The system of accountability calls for a paradigm shift which will require that all stakeholders focus on improving performance to achieve the target of eighty-five percent (85%) mastery in numeracy of the educable population by 2015. The performance of roles by all stakeholders has implications for rewards and sanctions to be determined using the School Accountability Matrix (SAM). This is the approach that has been mandated to hold all institutional stakeholders accountable for effective numeracy instruction in Jamaican schools at Grades 1- 6. The chain of accountability reflected in the School Accountability Matrix requires that each party contributes to the national target by ensuring the school's target is met through diligent application to duty.

The primary objective of the SAM for numeracy is to sensitize educational stakeholders to a set of core principles that will govern the system of accountability and provide a clearer picture of how it should be implemented. The central theme in the document is that a robust and effective culture of accountability is essential to sustainable solutions to numeracy achievement in Jamaica.

It is hoped that the SAM will serve as an effective framework for establishing and improving other accountability systems.

The SAM for numeracy consists of three components:

1. A Business Operation Plan that provides a framework to guide time-bound activities. For

each strategic function, the Business Operation Plan outlines the persons or units responsible and timelines for the completion of tasks and/ or deliverables.

2. An Accountability Matrix that will measure the achievements of process owners at all levels and provide a clear link between performance, rewards and sanctions. This Accountability Matrix will:

   a) Clearly articulate the Ministry of Education's national and school level numeracy targets for the primary age cohort. This is to ensure that all stakeholders' actions and decisions are aligned and consistent with the numeracy attainment targets stipulated in the RPC.

   b) Differentiate the performance of schools in meaningful ways so that appropriate support and interventions are given to those schools that are in need, and that the top-performing/high-growth schools are duly recognized as institutions of excellence.

3. A Numeracy Target Monitoring Tool (NTMT) that identifies internal and external factors that are favourable and/or unfavourable to achieving numeracy targets. It also provides diagnostic reviews to better link numeracy targets to meaningful support and interventions.

User Protocol

1. The School Accountability Matrix is to be disseminated by the end of September each year.

2. The Regional Director, with support from the Regional Numeracy Specialist, is responsible for ensuring the SAM is properly disseminated and filed as detailed below.

3. Individual SAMs should be filed with the following: The Board Chair, Principal, Regional Director, Regional Numeracy Coordinator/QEC Convener [2], National Numeracy Coordinator, DCEOs-Curriculum Support Services and Operations, CEO

4. Baseline data on numeracy proficiency should be collected by mid October each year and will serve to determine the value added by term at the end of each year

5. By the end of each term Numeracy Specialists are required to file a report with the relevant Regional Coordinator for review and assessment. (Report template is at Section B).

6. The National Numeracy Coordinator will provide the necessary guidance and make

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2   QEC Convener – Quality Education Circle Convener will over time replace the Regional Mathematics Coordinator as the conduit to the Regional Office and the Department of School Services under the transformed Ministry of Education.

recommendations for improvements or provide additional support where necessary, to ensure that the school's target will be met.

7. The SAM is a working document for the class teacher, Grade Coordinator and Numeracy Specialist

8. The school's Annual Numeracy Growth Rate is stated on the Matrix which is enclosed with this document.

9. The Annual Growth Rate is determined per school in order to satisfy the stated national target.

(Please see Appendix 2 of the SAM document)

School Improvement Planning

The School Improvement Plan for numeracy is a road map that articulates key elements necessary to impact the numeracy landscape in a school. Its design should promote and ensure consistency in the development of school wide numeracy operational plans. The plan must give emphasis to seven major tasks that educators must adhere to in developing a comprehensive and balanced numeracy programme. The seven major tasks are:

- **Instructional Leadership**
- **Curriculum Delivery**
- **Learning Environment**
- **Student Support**
- **Continuous Assessment**
- **Professional Development**
- **Home-School Partnership**

The School Improvement Plan for numeracy should be undertaken in two stages; the planning stage and the review stage. During the planning stage it is required that the activities, resources, means of verification, responsible agents and expected date (s) of achievement be documented for each major task. The review stage provides scope for reflection. It is at this stage that accomplishments and/or failures of major tasks are evaluated in a collaborative manner and further recommendations made. This document is intended to foster collaborative planning for numeracy improvement in a bid to achieve 85% numeracy of the educable population by 2015.

The Numeracy Target Monitoring Tool

The Numeracy Target Monitoring Tool (NTMT) is a companion document that forms a part of the system of accountability. The NTMT will be utilised by the National Numeracy Team and Regional Branch Offices in recognizing and tracking the progress of numeracy development across individual institutions with primary departments.

The goals of the NTMT are to:

- ensure that schools achieve annual numeracy targets;

- improve the quality of numeracy teaching and learning;

- provide assistance in identifying and resolving possible factors that may impede the attainment of numeracy targets;

- ensure the accuracy, validity, and reliability of data collection and data reporting.

The administration of the NTMT includes school visits and interviews with principals, teachers and students. Targeted and thorough school visits will be conducted by members of the National Numeracy Team and the Regional Branch Offices. These visits will collect information in six key areas:

- Demographic Information

- Student Support Services and Resources

- Numeracy Interventions

- Numeracy Professional Development

- Assessment

- Factors Affecting Performance

The information collected using the NTMT will be used in conjunction with the School Accountability Matrix for Numeracy to hold stakeholders (internal to the school environment) accountable for the numeracy performance of their schools. To this end, the tool will serve as concrete evidence of the efforts, initiatives and best practices being employed by individual institutions in an attempt to achieve their numeracy targets.